نام و نام خانوادگی ......محمدحسین .... پاسخنامه تشریحی تکلیف شماره ...۱... کلاس ...۲... B........

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$$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$$

$$\underbrace{1\quad 6\quad 0\quad 0\quad 0}_{7}\qquad \underbrace{2\quad 4\quad 0\quad 0\quad 0}_{6}$$

$$\boxed{2}$$

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الف) $\sqrt[4]{\frac{1}{(4+\sqrt7)}}\times\sqrt{1+\sqrt7}=\frac{1}{\sqrt[4]{(4+\sqrt7)}}\times\sqrt{1+\sqrt7}=\frac{\sqrt{(1+\sqrt7)}}{\sqrt[4]{(4+\sqrt7)}}=\sqrt[4]{\frac{\cancel{8+2\sqrt7}^{\,2}}{\cancel{4+\sqrt7}_{\,1}}}=\boxed{\sqrt[4]{2}}$

$\sqrt[2]{1+\sqrt7}=\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{8+2\sqrt7}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)=\frac{1}{\sqrt2}\times(-\sqrt2)=\boxed{-1}$

$\times\frac{\sqrt{10}-2}{\sqrt{10}-2}=\frac{\cancel{2\sqrt5}+5\sqrt2-2\sqrt2-\cancel{2\sqrt5}}{6}=\frac{3\sqrt2}{6}=\frac{\sqrt2}{2}=\frac{1}{\sqrt2}$ $\rightarrow C=\sqrt{9-5}=2,\ \cancel{\sqrt{\frac52}}-\sqrt{\frac12}-\cancel{\sqrt{\frac52}}-\sqrt{\frac12}=-\sqrt2$

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الف) $=\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}-\frac{2}{\sqrt3-1}=\sqrt2+\cancel{\sqrt3}-\cancel{\sqrt3}-1=\boxed{\sqrt2-1}$

$\rightarrow\times\frac{\sqrt3+1}{\sqrt3+1}=\frac{2\sqrt3+2}{2}=\sqrt3+1$

$\rightarrow\frac{(2\sqrt2+3\sqrt3)}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6}=\frac{10\sqrt2+15\sqrt3+4\sqrt3+9\sqrt2}{19}=\frac{19\sqrt2+19\sqrt3}{19}=\sqrt2+\sqrt3$

ب) $\frac{3\sqrt3-1}{4+\sqrt3}+\frac{1}{2-\sqrt3}=\frac{6\sqrt3-2-9+\sqrt3+4+\sqrt3}{8-3-4\sqrt3+2\sqrt3}=\frac{8\sqrt3-7}{5-2\sqrt3}\times\frac{5+2\sqrt3}{5+2\sqrt3}=\frac{40\sqrt3-35+48-14\sqrt3}{25-12}$

$=\frac{26\sqrt3+13}{13}=\boxed{2\sqrt3+1}$

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الف) $\frac{\overbrace{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}^{A}}{\sqrt{\sqrt3-\sqrt2}}-2\longrightarrow\sqrt6+2-2=\sqrt6$

$A^2=(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1})^2=1+\sqrt3+\sqrt3-1+2\sqrt{3-1}=2\sqrt3+2\sqrt2\longrightarrow A=\sqrt2(\sqrt{\sqrt3+\sqrt2})$

$\frac{\sqrt2(\sqrt{\sqrt3+\sqrt2})}{(\sqrt{\sqrt3-\sqrt2})}=\sqrt2\times\sqrt{\frac{\sqrt3+\sqrt2}{\sqrt3-\sqrt2}}\longrightarrow\frac{\sqrt3+\sqrt2}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2}=(\sqrt3+\sqrt2)^2\longrightarrow\sqrt2\times\sqrt{(\sqrt3+\sqrt2)^2}=\sqrt2(\sqrt3+\sqrt2)$

ب) $=2^{\frac{8}{12}}\times2\sqrt[4]{2}\times2\times2^{\frac{1}{12}}=2^{\frac23}\times3\times2^{\frac14}\times2\times2^{\frac{1}{12}}=2^{\frac{8}{12}}\times2^{\frac{3}{12}}\times2^{\frac{1}{12}}\times2\times3=2\times2\times3=\boxed{12}$

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$\rightarrow\frac{3^x(1+3+9+\cancel{27}^{\,364}+81+243)}{2^x(\frac14+\frac12+\cancel{1+2+4+8})_{\,15/75}}=52\Rightarrow\frac{3^x}{2^x}=\frac{15/75\times52}{364}=2/25=\frac94\Rightarrow x=\boxed{2}$

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$(a^2+b^2-2ab)^2=((a-b)^2)^2=(a-b)^4$, $(a^2+b^2+2ab)^2=((a+b)^2)^2=(a+b)^4 \Rightarrow ((a+b)(a-b))^4=(a^2-b^2)^4=((a^2-b^2)(a^2-b^2))^2=$ $(a^4+b^4-2a^2b^2)^2=(\sqrt{6}-\cancel{4}+\sqrt{6}+\cancel{4}-2\sqrt{3})^2=(2\sqrt{3}(\sqrt{3}-1))^2=2\sqrt{3}(\sqrt{3}-1)(2\sqrt{3}\times(\sqrt{3}-1))$ $=8\times(4-2\sqrt{3})=16(2-\sqrt{3})$ $t(2-\sqrt{3})=16(2-\sqrt{3}) \Rightarrow t=\boxed{16}$	۲	۶
$(a+\frac{1}{a}+\sqrt{2})^2(a+\frac{1}{a}-\sqrt{2})^2=(a^2+1-a\sqrt{2}+1+\frac{1}{a^2}-\frac{\sqrt{2}}{a}+a\sqrt{2}+\frac{\sqrt{2}}{a}-2)^2=(a^2+\frac{1}{a^2})^2$ $=a^4+\frac{1}{a^4}+2=2^t$ $a=\sqrt[4]{7-4\sqrt{3}}=\sqrt[4]{(2-\sqrt{3})^2}=\sqrt{2-\sqrt{3}} \to (2-\sqrt{3})^2+\frac{1}{(2-\sqrt{3})^2}+2=\frac{(2-\sqrt{3})^4+1+2(2-\sqrt{3})^2}{(2-\sqrt{3})^2}$ $\to \frac{a^2+2a+1}{a}=\frac{(a+1)^2}{a} \Rightarrow \frac{(8-4\sqrt{3})^2}{(2-\sqrt{3})^2}=\frac{16(2-\sqrt{3})^2}{(2-\sqrt{3})^2}=16=2^4=2^t \Rightarrow t=\boxed{4}$ $(8-4\sqrt{3})^2=64+48-64\sqrt{3}=112-64\sqrt{3}=16(2-\sqrt{3})^2$	۲	۷
$(28)^{-\frac{1}{3}}=\frac{1}{(28)^{\frac{1}{3}}}=\frac{1}{\sqrt[3]{28}} \xrightarrow{A=4} \frac{1}{\sqrt[3]{2\times 4}}=\boxed{\frac{1}{2}}$ $\sqrt[5]{4\sqrt[3]{16}}\times(\frac{1}{2})^{-\frac{4}{3}}=\sqrt[15]{2^{10}}\times\sqrt[15]{2^{20}}=\sqrt[15]{2^{30}}=2^2=4=A$ $2^2\times\sqrt[3]{2^4}=\sqrt[3]{2^{10}}$	۲	۸
$\sqrt[7]{a}=27a^{\frac{15}{7}} \Rightarrow \frac{\sqrt[7]{a}}{\sqrt[7]{a^{15}}}=27 \Rightarrow \sqrt[7]{\frac{a}{a^{15}}}=27 \Rightarrow \sqrt[7]{\frac{1}{a^{14}}}=27 \Rightarrow \frac{1}{a^2}=27 \Rightarrow a=\frac{1}{3\sqrt{3}}$ $\frac{\frac{1}{a}-3}{1+\sqrt{3}}=\frac{3\sqrt{3}-3}{1+\sqrt{3}}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{3(3+1-2\sqrt{3})}{3-1}=\frac{3(4-2\sqrt{3})}{2}=\frac{12-6\sqrt{3}}{2}=\boxed{6-3\sqrt{3}}$	۲	۹
$(\sqrt{x+a}-\sqrt{x-4})(\sqrt{x+a}+\sqrt{x-4})=2(\sqrt{x+a}+\sqrt{x-4}) \Rightarrow \cancel{x}+a-\cancel{x}+4=2\sqrt{x+a}+2\sqrt{x-4}$ $\Rightarrow a+4=2(\sqrt{x+a}+\sqrt{x-4}) \Rightarrow \frac{a+4}{2}=\sqrt{x+a}+\sqrt{x-4}$ $\Rightarrow \sqrt{x+a}+\sqrt{x-4}-2=\frac{a}{2}+2-2=\boxed{\frac{a}{2}}$	۲	۱۰