

رسم یکان

$$(1! + 2! + 3! + \dots + 20!) (2! + 4! + 6! + \dots + 24!)$$

از ۱۰ به بعد رسم یکان ناکند (۰)

$$(1+2)(2+4) \dots (20+40) = 1120 \quad \text{پ} \quad \text{پ}$$

الف) $\sqrt{4+\sqrt{5}}^{-1} \times \sqrt{1+\sqrt{5}} = 4\sqrt{\frac{1}{4+\sqrt{5}}} \times \sqrt{1+\sqrt{5}} = 4\sqrt{\frac{1+\sqrt{5}}{4+\sqrt{5}}} = 4\sqrt{2} \quad \text{پ}$

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{1}+2}\right) \left(\frac{\sqrt{3}-\sqrt{5}-\sqrt{2}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}\right) = \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} \times (-\sqrt{2}) = \frac{-\sqrt{2}}{\sqrt{3}-\sqrt{5}} = -1 \quad \text{پ}$

الف) $\frac{\sqrt{8}+\sqrt{20}}{5-\sqrt{5}} - 2(\sqrt{9}-1)^{-1} =$

$$\frac{(\sqrt{8}+\sqrt{20})(5+\sqrt{5}) - 2\sqrt{3}+2}{25-5} = \frac{5\sqrt{2}+10\sqrt{5}+5\sqrt{2}+10\sqrt{5}+9\sqrt{2}-\sqrt{3}+1-\sqrt{2}+\sqrt{3}-\frac{2}{2}}{19} = \frac{19\sqrt{2}-1}{19} = \sqrt{2}-\frac{1}{19} \quad \text{پ}$$

ب) $\frac{\sqrt{20}-1}{2+\sqrt{3}} + (2-\sqrt{3})^{-1} = \frac{\sqrt{20}-1}{2+\sqrt{3}} + \frac{2+\sqrt{3}}{1} = \frac{12\sqrt{3}-4-9+\sqrt{20}+2+\sqrt{3}-\sqrt{3}-1+2\sqrt{3}}{13} = \frac{2\sqrt{3}+1}{13} \quad \text{پ}$

الف) $A = \frac{\sqrt{1+\sqrt{3}}+\sqrt{3}-1}{\sqrt{3}-\sqrt{2}} \quad \text{پ}$

$$A^2 = \frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{2}(\sqrt{3}+\sqrt{2})}{\sqrt{3}-\sqrt{2}} = 2(\sqrt{3}+\sqrt{2})^2$$

ب) $A = \sqrt{2}(\sqrt{3}+\sqrt{2}) = 2+\sqrt{6} \quad \text{پ}$

ب) $\sqrt[4]{2} \times \sqrt[4]{12} \times \sqrt[4]{4} = \sqrt[4]{2 \times 12 \times 4} = \sqrt[4]{96} = 2\sqrt[4]{6} \quad \text{پ}$

ج) $\frac{10^4-1}{3-1} = \frac{9999}{2} = 4999.5 \quad \text{پ}$

الف) $\frac{3^{201} + 3^{201} + \dots + 3^{201} + 3}{3^{201-2} + 3^{201-1} + \dots + 3^{201} + 3} = 3^2 = 9 \quad \text{پ}$

ب) $\frac{3^{201} (1+3+3^2+3^3+\dots+3^{200})}{3^{201-2} (1+2+2^2+\dots+2^{200})} = \frac{3^{201} \times 3^{201}}{2^{201} \times 2^{201}} = \frac{3^{402}}{2^{402}} = \left(\frac{3}{2}\right)^{402} \quad \text{پ}$

ج) $\frac{3^{201} - 3}{3^{201} - 3} = 1 \quad \text{پ}$

$$a = \sqrt[4]{54 - r} \quad (a^4 + b^4 - 4ab)^2 \neq (a^2 + b^2 + 2ab)^2 = 4(r - \sqrt{r})$$

$$b = \sqrt[4]{54 + r} \quad \text{---}$$

$$(a^4 + b^4 - 4ab)^2 = (a^4 + b^4 - (a-b)^4)((a+b)^4) = (a^4 - b^4)^2$$

$$(\sqrt[4]{54 - r} - \sqrt[4]{54 + r} - 2\sqrt{r})^2 = (2\sqrt[4]{54 - r}\sqrt{r})^2 = 4r + 1 - 14\sqrt{r} - 32 - 14\sqrt{r} = 4(r - \sqrt{r})$$

$$= \boxed{t = 14}$$

$$a = \sqrt[4]{v - r\sqrt{r}} = (a + \frac{1}{a} + \sqrt{r})^2 (a + \frac{1}{a} - \sqrt{r})^2 = r^2$$

$$= ((a + \frac{1}{a})^2 - r)^2 - (a^2 + \frac{1}{a^2} + r - r)^2 = (a^2 + \frac{1}{a^2})^2 - a^4 - \frac{1}{a^4} + r^2$$

$$= v - r\sqrt{r} + \frac{1}{\sqrt{v - r\sqrt{r}}} + r^2 = 9 - 4\sqrt{r} + \frac{v + r\sqrt{r}}{49 - 4r} = 14 = r^2 \Rightarrow \boxed{r = 4}$$

$$A = \sqrt[4]{\frac{1}{r^3}} \left(\frac{1}{r}\right)^{-\frac{r}{4}} = r^{\frac{1}{4}} \cdot r^{\frac{r}{4}} = r^{\frac{1+r}{4}} = r$$

$$(rA)^{-\frac{1}{4}} = (1)^{-\frac{1}{4}} = (r^{\frac{1+r}{4}})^{-\frac{1}{4}} = r^{-\frac{1+r}{4}} = \boxed{\frac{1}{r}}$$

$$a^{\frac{1}{v}} \leq r v a^{\frac{1}{v}} \Rightarrow r v = a^{-\frac{1}{v}} = a^{-r} \Rightarrow \frac{1}{a^r} = r v \Rightarrow \boxed{a = r\sqrt{r}}$$

$$\frac{(\frac{1}{a} - r)}{(1 + \sqrt{r})} = \frac{r\sqrt{r} - r}{1 + \sqrt{r}} = \frac{r(\sqrt{r} - 1)(\frac{\sqrt{r} + 1}{\sqrt{r} + 1})}{(1 + \sqrt{r})(\frac{\sqrt{r} - 1}{\sqrt{r} - 1})} = \frac{r(r - r\sqrt{r})}{r} = \frac{12 - 4\sqrt{r}}{r}$$

$$= \boxed{4 - 2\sqrt{r}}$$

$$\sqrt{a + r} - \sqrt{a - r} = r \Rightarrow (\sqrt{a + r} - \sqrt{a - r})(\sqrt{a + r} + \sqrt{a - r}) = a + r - a + r = 2r$$

$$= a + r \Rightarrow \sqrt{a + r} + \sqrt{a - r} = \frac{a}{r} + r \Rightarrow \sqrt{a + r} + \sqrt{a - r} - r = \frac{a}{r} \Rightarrow \boxed{\frac{a}{r}}$$