نام و نام خانوادگی ..پویا ... پاسخنامه تشریحی تکلیف شماره ..۲۱... کلاس ..دهم ریاضی....

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رقم یکان

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$

$1,\ 6,\ 0,\ 0,\ 0 \qquad 2,\ 24,\ 0,\ 0$

از !۵ به بعد رقم یکان فاکتوریل ۵۴ ←

$(1+6)(2+24)=7\times26=182 \rightarrow 2$

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الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt{1+\sqrt7}=\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}}=\sqrt[4]{2}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\underbrace{\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}}_{m}\right)=\frac{\sqrt2+\sqrt5}{\sqrt2(\sqrt2+\sqrt5)}\times(-\sqrt2)=\frac{-\sqrt2}{\sqrt2}=-1$

$m^2=3-\sqrt5+3+\sqrt5-2\sqrt{9-5}=2$ ، $m<0 \rightarrow m=-\sqrt2$

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الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}-2(\sqrt[4]{9}-1)^{-1}=\frac{(\sqrt8+\sqrt{27})(5+\sqrt6)}{\cancel{25-6}\ 19}-\frac{2\sqrt3+2}{2}$

$=\frac{10\sqrt2+15\sqrt3+4\sqrt3+9\sqrt2}{19}-\sqrt3-1=\sqrt2+\sqrt3-\sqrt3-1=\sqrt2-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}+(\sqrt3-2)^{-1}$ ، $\frac{(3\sqrt3-1)(4-\sqrt3)}{13}$ ، $\frac{2+\sqrt3}{1}$

$=\frac{12\sqrt3-4-9+\sqrt3}{13}+2+\sqrt3=\sqrt3-1+2+\sqrt3=2\sqrt3+1$

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الف) $\underbrace{\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}}_{A}-2=2+\sqrt6-2=\sqrt6$

$A^2=\frac{1+\sqrt3+\sqrt3-1+2\sqrt2}{\sqrt3-\sqrt2}=\frac{2(\sqrt3+\sqrt2)(\sqrt2+\sqrt3)}{1}=2(\sqrt3+\sqrt2)^2$

$A=\sqrt2(\sqrt3+\sqrt2)=2+\sqrt6$

ب) $\sqrt[4]{4\sqrt[3]{32}}\times\sqrt[4]{162}\times\sqrt{4\sqrt3}$

$=2^{\frac{7}{12}}\times3\times2^{\frac14}\times2^{\frac{13}{12}}=3\times2^2=12$

جمع دنباله هندسی $=\frac{3^6-1}{3-1}=\frac{728}{2}=364$

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$\frac{3^n+3^{n+1}+3^{n+2}+\dots+3^{n+5}}{2^{n-2}+2^{n-1}+2^n+\dots+2^{n+3}}=52 \rightarrow \frac{3^n(1+3+3^2+3^3+3^4+3^5)}{2^{n-2}\underbrace{(1+2+2^2+\dots+2^5)}_{\frac{2^6-1}{2-1}=63}}=\frac{3^n\times364}{2^{n-2}\times63}=52$

$\frac{3^n\times\cancel{364}^{\,52}}{2^{n-2}\times\cancel{63}_{\,9}}=52 \Rightarrow \frac{3^n}{2^{n-2}}=9 \Rightarrow 3^n=9,\ 2^{n-2}=1 \Rightarrow n=2$

$a = \sqrt[4]{\sqrt{6}-2} \quad (a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = t(2-\sqrt{3})$

$b = \sqrt[4]{\sqrt{6}+2}$

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = ((a-b)^2)^2((a+b)^2)^2 = (a^2-b^2)^4$ ۶

$(\sqrt{6}-2+\sqrt{6}+2-2\sqrt{2})^2 = (2\sqrt{6}-2\sqrt{2})^2 = 24+8-16\sqrt{3} = 32-16\sqrt{3} = t(2-\sqrt{3})$

$\Rightarrow t = 16$

$a = \sqrt[4]{7-4\sqrt{3}} \quad (a+\frac{1}{a}+\sqrt{2})^2(a+\frac{1}{a}-\sqrt{2})^2 = 2^t$

$= ((a+\frac{1}{a})^2-2)^2 = (a^2+\frac{1}{a^2}+2-2)^2 = (a^2+\frac{1}{a^2})^2 = a^4+\frac{1}{a^4}+2$ ۷

$= 7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2 = 9-4\sqrt{3}+\frac{7+4\sqrt{3}}{49-48} = 16 = 2^t \Rightarrow t = 4$

$A = \sqrt[5]{2^3\sqrt{16}} \quad (\frac{1}{2})^{-\frac{4}{3}} = 2^{\frac{20}{15}} \cdot 2^{\frac{4}{3}} = 2^2 = 4$

$(2A)^{-\frac{1}{3}} = (8)^{-\frac{1}{3}} = (2^3)^{-\frac{1}{3}} = 2^{-1} = \frac{1}{2}$ ۸

$a^{\frac{1}{7}} = 27\, a^{\frac{15}{7}} \Rightarrow 27 = a^{-\frac{14}{7}} = a^{-2} \Rightarrow \frac{1}{a^2} = 27 \Rightarrow a = \frac{1}{3\sqrt{3}}$

$\frac{(\frac{1}{a}-3)}{(1+\sqrt{3})} = \frac{3\sqrt{3}-3}{1+\sqrt{3}} = \frac{3(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3(4-2\sqrt{3})}{2} = \frac{12-6\sqrt{3}}{2}$ ۹

$= 6-3\sqrt{3}$

$\sqrt{x+a}-\sqrt{x-4} = 2 \Rightarrow (\underbrace{\sqrt{x+a}-\sqrt{x-4}}_{2})(\sqrt{x+a}+\sqrt{x-4}) = x+a-x+4$

$= a+4 \Rightarrow \sqrt{x+a}+\sqrt{x-4} = \frac{a}{2}+2 \Rightarrow \sqrt{x+a}+\sqrt{x-4}-2 = \frac{a}{2}+2-2 = \frac{a}{2}$ ۱۰