ریاضی دهم سری B

۱- $(1!+3!+5!+\dots+25!)(2!+4!+\dots+26!)$

یکان → $2!+4! = 26 \to (6)$

$1!+3! = 7$

جزء آخر → $7\times6=42 \to (2)$

۲- الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt{1+\sqrt7} = \sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt[4]{8+2\sqrt7} = \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{2}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \left(\frac{\sqrt2+\sqrt5}{\sqrt2(\sqrt5+\sqrt2)}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) \Rightarrow$

$\frac{\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}}{\sqrt2}\times\frac{\sqrt2}{\sqrt2} = \frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2} = \frac{\sqrt{(\sqrt5-1)^2}-\sqrt{(\sqrt5+1)^2}}{2} = \frac{\sqrt5-1-\sqrt5-1}{2}$

$= \frac{-2}{2} = -1$

۳- الف) $\frac{2\sqrt2+3\sqrt3}{5-\sqrt6} - 2(\sqrt[4]{9}-1)^{-1} = \frac{2\sqrt2+3\sqrt3}{(\sqrt2)^2+(\sqrt3)^2-\sqrt3\times\sqrt2} - \frac{2}{\sqrt3-1} =$

$\frac{\frac{(2\sqrt2+3\sqrt3)(\sqrt2+\sqrt3)}{(\sqrt2)^3+(\sqrt3)^3-(\sqrt3\times\sqrt2)(\sqrt2+\sqrt3)}}{2\sqrt2+3\sqrt3} - 2\times\frac{\sqrt3+1}{3-1} = \sqrt2+\sqrt3-\sqrt3-1 = \sqrt2-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1} = \frac{3\sqrt3-1}{1+3-\sqrt3}\times\frac{\sqrt3-1}{\sqrt3-1}+\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = \frac{(3\sqrt3-1)(\sqrt3-1)}{(3\sqrt3-1)}+2+\sqrt3 =$

$\sqrt3-1+2+\sqrt3 = 2\sqrt3+1$

۴- الف) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}-2 \times\frac{\sqrt{\sqrt3+\sqrt2}}{\sqrt{\sqrt3+\sqrt2}} = (\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1})(\sqrt{\sqrt3-\sqrt2})-2$

$\Rightarrow \sqrt{[(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1})(\sqrt{\sqrt3-\sqrt2})]^2} = \sqrt{(2\sqrt3-2\sqrt2)(\sqrt3-\sqrt2)} = \sqrt2(\sqrt3-\sqrt2) \Rightarrow \sqrt2(\sqrt3-\sqrt2)-2$

$= \sqrt6-4$

ب) $\sqrt[4]{\sqrt[3]{28}}\times\sqrt[4]{162}\times\sqrt{4\sqrt2} = 2^{\frac23}\times2^{\frac14}\times3\times2\times2^{\frac14} = 3\times4 = 12$

۵-

$$\frac{۳^x(۱+۳+۹+۲۷+۸۱+۲۴۳)}{۲^x(\frac{۱}{۴}+\frac{۱}{۲}+۱+۲+۴+۸)} = ۵۴ \Rightarrow ۳^x \times ۳۶۴ = ۲^x \times ۸۱۹ \Rightarrow ۲^x \times ۲^۲ \times ۹۱ = ۳^x \times ۳^۲ \times ۹۱$$

$$\Rightarrow ۲^{x-۲} = ۳^{x-۲} \Rightarrow x-۲=۰ \Rightarrow \boxed{x=۲}$$

۶-

$$(a^۲+b^۲-۲ab)^۲(a^۲+b^۲+۲ab)^۲ = (a^۴+b^۴-۲a^۲b^۲)^۲ = \left[(\sqrt[۴]{\sqrt{۶}-۲})+(\sqrt[۴]{\sqrt{۶}+۲})-۲(\sqrt{\sqrt{۶}-۲})(\sqrt{\sqrt{۶}+۲})\right]^۲$$

$$= (۲\sqrt{۶}-۲\sqrt{۲})^۲ = (۲\sqrt{۲}(\sqrt{۳}-۱))^۲ = ۱۶(۴-۲\sqrt{۳}) = t(۴-۲\sqrt{۳}) \Rightarrow t=۱۶$$

۷

$$(a+\frac{۱}{a}+\sqrt{۲})(a+\frac{۱}{a}-\sqrt{۲}) = (a^۲+\frac{۱}{a^۲}-۲+۲)^۲ = a^۴+\frac{۱}{a^۴}+۲ = ۲^t$$

$$\star\ a = \sqrt[۴]{۷-۴\sqrt{۳}} = \sqrt{۲-\sqrt{۳}} \Rightarrow (a^۲+\frac{۱}{a^۲})^۲ = (۲-\sqrt{۳}+\frac{۱}{۲-\sqrt{۳}})^۲$$

$$= (۲-\sqrt{۳}+۲+\sqrt{۳})^۲ = ۱۶ = ۲^t \Rightarrow t=۴$$

۸-

$$A = \sqrt[۵]{۴\sqrt[۳]{۱۶}} \times ۲^{\frac{۴}{۳}} = ۲^{\frac{۲}{۳}} \times ۲^{\frac{۴}{۳}} = ۴ \Rightarrow (۲A)^{-\frac{۱}{۳}} = (\frac{۱}{۸})^{\frac{۱}{۳}} = (\frac{۱}{۲})$$

۹-

$$۲۷a^{\frac{۱۵}{۷}} = a^{\frac{۱}{۷}} \Rightarrow ۲۷a^۲ = ۱ \Rightarrow a = \frac{۱}{۳\sqrt{۳}}$$

$$\Rightarrow \frac{\frac{۱}{a}-۳}{۱+\sqrt{۳}} = \frac{۳\sqrt{۳}-۳}{۱+\sqrt{۳}} \times \frac{\sqrt{۳}-۱}{\sqrt{۳}-۱} = \frac{۳(۴-۲\sqrt{۳})}{۲} = ۶-۳\sqrt{۳}$$

۱۰

$$(\sqrt{x+a}-\sqrt{x-۴})(\sqrt{x+a}+\sqrt{x-۴}) = a+۴ \rightarrow \sqrt{x+a}+\sqrt{x-۴} = \frac{a+۴}{۲}$$

$$\Rightarrow \sqrt{x+a}+\sqrt{x-۴}-۲ = \frac{a}{۲}+۲-۲ = \frac{a}{۲}$$