

(21)

بهره

ایستادن

19

$$(1+4+0+0+...+0) \times (2+2+0+0+...+0) = 10 \times 4 = 40$$

①

(2)

$$\sqrt[4]{(x+\sqrt{x})^{-1}} \sqrt{x+\sqrt{x}} = (x+\sqrt{x})^{-\frac{1}{4}} (x+\sqrt{x})^{\frac{1}{4}} = \frac{(x+\sqrt{x})^{\frac{1}{4}}}{(x+\sqrt{x})^{\frac{1}{4}}} \quad (\text{انف})$$

$$\Rightarrow \left(\frac{(x+\sqrt{x})}{x+\sqrt{x}} \right)^{\frac{1}{4}} = x^{\frac{1}{4}} \cdot \sqrt[4]{x} \quad \checkmark$$

(3)

(4)

$$\sqrt{x+\sqrt{x}} = \frac{\sqrt{x}+1}{\sqrt{x}} = \frac{\sqrt{x}-1}{\sqrt{x}} \cdot \frac{\sqrt{x}+1}{\sqrt{x}} = \frac{\sqrt{x}-1}{\sqrt{x}} \cdot \frac{\sqrt{x}+1}{\sqrt{x}}$$

$$\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}-1} \times (-\sqrt{x}) = \frac{(-1) \sqrt{x+\sqrt{x}}}{\sqrt{x}-1} = \frac{\sqrt{x}+\sqrt{x}}{\sqrt{x}-1} \times \frac{\sqrt{x}+\sqrt{x}}{\sqrt{x}+\sqrt{x}} = \frac{10\sqrt{x}+10\sqrt{x}}{19} = \frac{19\sqrt{x}+19\sqrt{x}}{19} = \frac{\sqrt{x}+\sqrt{x}}{1} \quad (\text{انف})$$

(5)

$$(\sqrt{x}-1)^{-1} = \frac{1}{\sqrt{x}-1} \times \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{1}{\sqrt{x}+1} (\sqrt{x}+1) \rightarrow (\sqrt{x}+\sqrt{x}) - 2\left(\frac{1}{\sqrt{x}+1}\right) = \sqrt{x}-1$$

$$\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}-1} = \frac{-2(\sqrt{x}-1)^{-1}}{\sqrt{x}-1} = \frac{2\sqrt{x}+2\sqrt{x}}{\sqrt{x}-1} = \frac{2}{\sqrt{x}+1} \Rightarrow \frac{2\sqrt{x}+2\sqrt{x}}{\sqrt{x}-1} \times \frac{\sqrt{x}+1}{\sqrt{x}+1}$$

(6)

$$\Rightarrow \frac{19(\sqrt{x}+\sqrt{x})}{19} = \sqrt{x}+\sqrt{x}$$

$$\frac{2\sqrt{x}-1}{\sqrt{x}+1} + \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{x}+1} \times \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{1}{\sqrt{x}+1}$$

$$\Rightarrow \frac{\sqrt{x}+\sqrt{x}}{1} = \frac{2\sqrt{x}-1}{\sqrt{x}+1} \times \frac{\sqrt{x}-1}{\sqrt{x}-1} = 1-\sqrt{x}$$

$$\frac{\sqrt{x}-1}{\sqrt{x}+1} = \frac{\sqrt{x}-1}{\sqrt{x}+1} \times \frac{\sqrt{x}-1}{\sqrt{x}-1} = \frac{10\sqrt{x}-9-2\sqrt{x}}{13} = \frac{13\sqrt{x}-13}{13} = \sqrt{x}-1$$

$$2+\sqrt{x}+1-\sqrt{x} = 3 \quad (\text{انف}) \quad (\sqrt{x})^{-1} = \frac{1}{\sqrt{x}} \times \frac{\sqrt{x}+1}{\sqrt{x}+1} = \sqrt{x}+1 \rightarrow \sqrt{x}-1+2+\sqrt{x} = 2\sqrt{x}+1$$

(مطلوب)

①

$$r^n (1 + r + r^2 + \dots + r^{n-1})$$

$$r^n (r^{-1} + r^0 + r^1 + \dots + r^{n-1}) = \frac{r^n}{r} (1 + r + r^2 + \dots + r^{n-1}) = r^{n-1} (1 + r + r^2 + \dots + r^{n-1})$$

$$((a-b)^r)^r ((a+b)^r)^r = (a-b)^r (a+b)^r = (a^2 - b^2)^r \Rightarrow a^r b^r \sqrt{a^2 - b^2} = \sqrt{a^2 - b^2}$$

$$\Rightarrow (\sqrt{a} - b) + (\sqrt{a} + b) = 2\sqrt{a} \Rightarrow (\sqrt{a} - b)(\sqrt{a} + b) = a - b^2 \Rightarrow 2\sqrt{a} = \sqrt{a^2 - b^2} \Rightarrow 4a = a^2 - b^2 \Rightarrow a^2 - 4a - b^2 = 0$$

$$(1 + \frac{1}{a})^r = 1 + \frac{r}{a} + \frac{r(r-1)}{2!} \frac{1}{a^2} + \dots = (1 + \frac{1}{a})^r \Rightarrow \sqrt{1 + \frac{1}{a}} = \sqrt{1 + \frac{1}{a}} = 1 + \frac{r}{2a} + \dots$$

$$\Rightarrow \frac{1}{a^r} = 1 + \frac{r}{a} + \frac{r(r-1)}{2!} \frac{1}{a^2} + \dots = (1 + \frac{1}{a})^r \Rightarrow \frac{1}{a^r} = (1 + \frac{1}{a})^r$$

$$(x+y)^r (x-y)^r = (x^2 - y^2)^r$$

$$\frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} = \frac{1}{r^3} \Rightarrow \frac{1}{r} \times \frac{1}{r} = \frac{1}{r^2} \Rightarrow \frac{1}{r} = \frac{1}{r^2} \Rightarrow r = 1$$

$$(A)^{-\frac{1}{r}} = (A^{\frac{1}{r}})^{-1} = \frac{1}{A^{\frac{1}{r}}}$$

$$\frac{1}{a^r} = \frac{1}{a^r} \Rightarrow \frac{1}{a^r} = \frac{1}{a^r}$$

$$\Rightarrow 2a^r - a = a(2a^{r-1} - 1) = 0 \Rightarrow 2a^{r-1} - 1 = 0 \Rightarrow 2a^{r-1} = 1 \Rightarrow a^{r-1} = \frac{1}{2} \Rightarrow a = \sqrt[r-1]{\frac{1}{2}}$$

$$A = \sqrt{a+b} \quad B = \sqrt{a-b} \quad A-B = \sqrt{a+b} - \sqrt{a-b} \Rightarrow (A-B)(A+B) = A^2 - B^2 = a+b - (a-b) = 2b$$

$$\Rightarrow 2(A+B) = \frac{2b}{A+B} \Rightarrow (A+B)^2 = \frac{b}{A+B} \Rightarrow A+B = \sqrt{\frac{b}{A+B}} \Rightarrow A+B = \sqrt{\frac{b}{A+B}}$$

سؤال ٣

$$\text{الف) } \frac{\overbrace{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}^A}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2 \rightarrow \sqrt{6} + 2 - 2 = \sqrt{6}$$

$$A^2 = (\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1})^2 = 1 + \sqrt{3} + \sqrt{3}-1 + 2\sqrt{\sqrt{3}-1} = 2\sqrt{3} + 2\sqrt{2} \rightarrow A = \sqrt{2}(\sqrt{\sqrt{3}+1})$$

$$\frac{\sqrt{2}(\sqrt{\sqrt{3}+1})}{(\sqrt{\sqrt{3}-1})} = \sqrt{2} \times \sqrt{\frac{\sqrt{3}+1}{\sqrt{3}-1}} \rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = (\sqrt{3}+1)^2 \rightarrow \sqrt{2} \times \sqrt{(\sqrt{3}+1)^2} = \sqrt{2}(\sqrt{3}+1)$$

$$\begin{aligned} \text{ب) } \sqrt[4]{\sqrt[3]{18}} \times \sqrt[4]{12} \times \sqrt[4]{\sqrt[3]{2}} &= (12)^{\frac{1}{12}} \times \sqrt[4]{12 \times 18} \times \sqrt[4]{\sqrt[3]{2}} = 12^{\frac{1}{12}} \times 12^{\frac{1}{4}} \times 2^{\frac{1}{12}} \\ &= 12^{\frac{1}{12}} \times 12^{\frac{1}{4}} \times 2^{\frac{1}{12}} = 12^{\frac{1}{12} + \frac{1}{4} + \frac{1}{12}} = 12^{\frac{2}{3}} = 12^{\frac{2}{3}} = 12 \end{aligned}$$

$$\sqrt[n]{a} = n \times a^{\frac{1}{n}} \rightarrow \sqrt[n]{a} = n \times \sqrt[n]{a^{\frac{1}{n}}} \rightarrow a^{\frac{1}{n}} = \frac{1}{n} \rightarrow a = \frac{1}{\sqrt[n]{n}} = \frac{1}{n\sqrt[n]{n}}$$

$$\rightarrow \left(\frac{1}{a} - n\right) = n\sqrt[n]{n} - n = n(\sqrt[n]{n} - 1) \rightarrow \frac{n(\sqrt[n]{n} - 1)}{\sqrt[n]{n} + 1} \times \frac{\sqrt[n]{n} - 1}{\sqrt[n]{n} - 1} = \frac{n(1 + \sqrt[n]{n} - n\sqrt[n]{n})}{n - 1} = \frac{n(1 - n\sqrt[n]{n})}{n - 1}$$

سؤال ٤

$$n(1 - n\sqrt[n]{n}) = 4 - 4\sqrt[4]{4}$$