

$$S = \frac{b}{a} \text{ و } \rho = \frac{c}{a} \rightarrow x^2 - 2x + 1 = 0 \rightarrow \begin{cases} S = \alpha + \beta = \frac{b}{a} = 0 \quad (1) \\ \rho = \alpha\beta = \frac{c}{a} = 1 \quad (2) \end{cases}$$

$$A = \frac{4\alpha + 8\beta}{\alpha\beta^2} \xrightarrow{\times \frac{\beta}{\beta}} \frac{4\alpha\beta + 8\beta^2}{\alpha\beta^3} \xrightarrow{(2) \alpha\beta=1} A = \frac{4 + 8\beta}{\beta^3} \rightarrow A = \frac{1}{\beta} \left(\frac{4}{\beta^2} + 8\beta \right) (*)$$

$$(*) \rightarrow \alpha\beta = 1 \rightarrow \alpha = \frac{1}{\beta} \rightarrow \alpha^3 = \frac{1}{\beta^3} \rightarrow A = \frac{1}{\beta} \left(\alpha^3 + 8\beta^3 \right) \xrightarrow{\alpha^3 + 8\beta^3 = S^3 - 3\alpha\beta S} A = \frac{1}{\beta} (S^3 - 3\alpha\beta S)$$

$$\rightarrow A = \frac{1}{\beta} (12 - 3 \times 1 \times 0) = \frac{1}{\beta} (12 - 0) = \frac{1}{\beta} \times 12 = 12 \rightarrow A = 12 \leftarrow \text{جواب}$$

چون هر دو نقطه $y = -4$ دارند می توانیم نتیجه بگیریم که محور تقارن دایره عمود بر خط $y = -4$ است

$$\frac{3 + (-1/2)}{2} = \frac{1/2}{2} = 0.125 = x_s$$

$$x_s = \frac{\alpha + \beta}{2} = 0.125 \rightarrow \alpha + \beta = 0.25$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{a} \xrightarrow{a=1} |\alpha - \beta| = \sqrt{\Delta} \rightarrow \left| \frac{12}{2} k \right| = \sqrt{b^2 - 4ac} \rightarrow \left| \frac{6}{1} k \right| = \sqrt{4k^2 - 20}$$

$$\xrightarrow{\text{مربع میزنیم}} \frac{36}{1} k^2 = 4k^2 - 20 \rightarrow \frac{20}{9} k^2 = 20 \rightarrow k^2 = 20 \times \frac{9}{20} \rightarrow k^2 = 9$$

$$\rightarrow \frac{k^2}{2} = \frac{9}{2} = 4.5 \rightarrow \left[\frac{k^2}{2} \right] = [4.5] = 4 \rightarrow \left[\frac{k^2}{2} \right] = 4$$

جواب: $\boxed{4}$

$$x_s = \frac{3 + (-1/2)}{2} = \frac{5}{4} = 1.25 \rightarrow \frac{-b}{2a} = -1 \rightarrow b = 2a \quad y_s = \frac{-1}{2a} = \frac{4ac - b^2}{4a^2} = \frac{4ac - 4a^2}{4a^2} = \frac{c - a}{a} = -1 \rightarrow c = a - 1$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-2a}{a} = -2 \rightarrow (\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \rightarrow 2\alpha\beta = -1 \rightarrow \alpha\beta = -\frac{1}{2} = \frac{c}{a}$$

$$y = a\alpha^2 + 2a\alpha + a + 1 \xrightarrow{\alpha=0} y = a + 1 \xrightarrow{\alpha=-\frac{1}{2}} y = a + 1 + \frac{a}{4} = \frac{5a}{4} + 1 = \frac{1}{4}$$

جواب: $\boxed{\frac{1}{4}}$

$$= x^2 + 2x + m = 0 \xrightarrow{x-1} x^2 - 2x - m = 0 \rightarrow \begin{cases} 1) \text{ برای اینکه دو ریشه داشته باشد باید } \Delta > 0 \\ 2) \text{ باید مقدار } m \text{ را در } x = \frac{9}{2} \text{ بدست آوریم} \end{cases}$$

$$\rightarrow \Delta > 0 \rightarrow b^2 - 4ac > 0 \rightarrow 4 + 4m > 0 \rightarrow 4m > -4 \rightarrow m > -1$$

$$\xrightarrow{x = \frac{9}{2}} \left(\frac{9}{2} \right)^2 - 2 \left(\frac{9}{2} \right) - m = 0 \rightarrow \frac{81}{4} - 9 - m = 0 \rightarrow \frac{81 - 36}{4} = m \rightarrow m = \frac{45}{4}$$

باید که m بزرگتر از $\frac{45}{4}$ باشد تا هر دو ریشه معادله کوچکتر از $\frac{9}{2}$ باشند

$$\begin{cases} 1) m > -1 \\ 2) m < \frac{45}{4} \end{cases} \rightarrow \text{مقدار } m \text{ باید در } (-1, \frac{45}{4}) \text{ باشد}$$

جواب: $\boxed{(-1, \frac{45}{4})}$

$\min \begin{cases} x_s & \text{چون گفته شده برای} \\ y_s & \text{کمترین مقدار است} \\ & \text{در حالت آن روش بالاست} \\ & \text{و ضریب } x \text{ مثبت است.} \end{cases} m > 0 \rightarrow x = \frac{-b}{ra} = \frac{-(-12)}{12m} = \frac{2}{m} \rightarrow y = mx^2 - 12x + 2m - 1$
 $m=3 \rightarrow x = \frac{-b}{ra} = \frac{-(-12)}{12 \cdot 3} = 1 \rightarrow x_s = 1$

$\rightarrow y = m(\frac{2}{m})^2 - 12 \times \frac{2}{m} + 2m - 1 = \frac{4}{m} - \frac{24}{m} + 2m - 1 = \frac{-20}{m} + 2m - 1 = y_{min} = 1$
 $\rightarrow \frac{-20}{m} + 2m - 1 = 1 \rightarrow -20 + 2m^2 - 2m = 0 \rightarrow 2m^2 - 2m - 20 = 0 \rightarrow \Delta = b^2 - 4ac = (-2)^2 - 4(2)(-20) = 4 + 160 = 164$
 $\rightarrow m = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-2) \pm \sqrt{164}}{4} = \frac{2 \pm 12.6}{4}$
 $m = \frac{10}{2} = 5 \rightarrow m = 5 \rightarrow x = \frac{2}{5} = 0.4$
 $m = \frac{-10}{2} = -5 \rightarrow m = -5 \rightarrow x = \frac{2}{-5} = -0.4$

$\alpha x \beta = a^r \rightarrow \alpha = a^r \rightarrow \frac{c}{a} = a^r \rightarrow \frac{c}{1} = a^r \rightarrow c = a^r \rightarrow |a-1| = a^r$
 $\rightarrow a^r - |a+1| = 0 \rightarrow (a-1)^r = 0 \rightarrow a = 1$
 $(a-1)^r = 0 \rightarrow a = 1$

$x^2 - vx - d = 0 \xrightarrow{x=t} t^2 - vt - d = 0 \rightarrow \rho = \frac{c}{a} = -d < 0$
 $\left. \begin{aligned} x^2 = t_1 > 0 &\rightarrow x_{1,2} = \pm \sqrt{t_1} \\ x^2 = t_2 < 0 &\rightarrow x \in \mathbb{R} \end{aligned} \right\} \rightarrow \begin{aligned} x_1 + x_2 &= 0 \rightarrow S = 0 \\ \rho = x_1 x_2 &= \sqrt{t_1} \times (-\sqrt{t_1}) = -t_1 \Rightarrow \rho^r = t_1^r \end{aligned}$
 $t^2 - vt - d = 0 \rightarrow t = \frac{v \pm \sqrt{v^2 + 4d}}{2} = \frac{v \pm \sqrt{64}}{2} \rightarrow t_1 = \frac{v + \sqrt{64}}{2} \rightarrow \rho^r = \frac{1}{2}(64 + 64 + 16\sqrt{64})$
 $\rightarrow 2\rho^r - 3\rho S + 2S = 2(64 + 64 + 16\sqrt{64}) - 0 + 0 = 2(128 + 128\sqrt{64})$
 $\rightarrow 2\rho^r - 3\rho S + 2S = 2(128 + 128\sqrt{64})$

$y = kx^2 - fx - g \rightarrow x = \frac{-b}{2a} = \frac{-(-f)}{2k} = \frac{f}{2k} = \frac{f}{k}$
 $y = kx^2 - fx - g \xrightarrow{x=\frac{f}{k}} y = k(\frac{f}{k})^2 - f \times \frac{f}{k} - g = \frac{f^2}{k} - \frac{f^2}{k} - g = \frac{f^2}{k} - g$
 $\rightarrow \frac{f^2}{k} + \frac{f^2}{k} = f^2 + g \rightarrow \frac{f^2}{k} = f^2 \rightarrow 2k = f \rightarrow k = \frac{f}{2}$
 $\rightarrow y = \frac{f^2}{k} - g \xrightarrow{k=\frac{f}{2}} y = \frac{f^2}{\frac{f}{2}} - g = 2f - g = -1 \rightarrow y_s = -1$

$-mx^2 + m(x+1) = -m - x \rightarrow -mx^2 + (m+1)x + (m+1) = 0 \rightarrow mx^2 - (m+1)x - (m+1) = 0$
 $\Delta \leq 0 \rightarrow b^2 - 4ac \leq 0 \rightarrow (m+1)^2 - 4(-m)(-m-1) \leq 0 \rightarrow m^2 + 2m + 1 - 4(-m^2 - m) \leq 0$
 $\rightarrow (m+1)^2 + 4m(m+1) \leq 0 \rightarrow (m+1)(m+1+4m) \leq 0 \rightarrow (m+1)(5m+1) \leq 0$
 $\rightarrow (m+1)(5m+1) \leq 0 \rightarrow (m+1)(m+1) \leq 0 \rightarrow -1 < m < -\frac{1}{5}$
 $-1 < m < -\frac{1}{5}$