سروش اخوان ۲۰ تکلیف ۲۲ دهم ب B

۱- $\frac{4\alpha+\beta^5}{5\beta^2} = \frac{\alpha^3\beta^2+\beta^5}{5\beta^2} = \frac{\beta^2(\alpha^3+\beta^3)}{5\beta^2} = \frac{\alpha^3+\beta^3}{5}$

$= \frac{S^3-3PS}{5}$ $\quad S=5,\ P=2 \Rightarrow \frac{125-30}{5} = 19$ ✓ ۲

۲- صفرهای تابع = جمع ریشه‌ها $= \frac{-b}{a}$

جمع نقاط هم عرض ثابت است $\Rightarrow -1/5+3 = 1/5$ ✓ ۲

۳- $x^2+2Kx+5=0 \qquad \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{4K^2-20}}{|a|} = \frac{4}{3}K$

$\Rightarrow 4K^2-20 = \frac{16}{9}K^2 \Rightarrow \frac{20K^2}{9} = 20 \Rightarrow K^2 = 9$ ۲

$\Rightarrow \left[\frac{K^2}{2}\right] = [4/5] = 4$ ✓

۴- $S = \frac{-b}{a} = -5+3 = -2 \Rightarrow b = 2a$

$\alpha^2+\beta^2 = 5 = S^2-2P \Rightarrow 4-2P = 5,\ P = \frac{c}{a} = -\frac{1}{2} \Rightarrow a = -2c$

$y = \frac{-\Delta}{4a} = \frac{-(b^2-4ac)}{4a} = \frac{-(2a)^2+4ac}{4a} = 1 \Rightarrow \begin{cases}-a+c=1\\ a=-2c\end{cases} \Rightarrow c = \frac{1}{3}$

$\left|\begin{matrix}0\\ \frac{1}{3}\end{matrix}\right. \rightarrow$ جواب ✓ ۲

۵- $x^2-5x-m=0 \qquad \Delta>0 \Rightarrow 25+4m>0 \Rightarrow m > \frac{-25}{4}$

$\frac{-b\pm\sqrt{\Delta}}{2a} < \frac{9}{2} \Rightarrow \frac{5\pm\sqrt{25+4m}}{2} < \frac{9}{2}$

$\Rightarrow 5\pm\sqrt{25+4m} < 9 \Rightarrow \pm\sqrt{25+4m} < 4 \Rightarrow 25+4m < 16 \Rightarrow 4m < -9 \Rightarrow m < -\frac{9}{4}$

$m > \frac{-25}{4} \wedge m < -\frac{9}{4} = \frac{-25}{4} < m < \frac{-9}{4}$ $\xrightarrow{\text{شامل}}$ $\{-6, -5, -4, -3\}$

۴ عدد صحیح ✓ ۲

۶- $\frac{-b}{2a} = \frac{12}{2m} = \frac{6}{m}$ ، min دار $\Rightarrow a > 0 \Rightarrow m > 0$

$\frac{36}{m} - \frac{72}{m} + 5m - 1 = 2 \Rightarrow 5m - 3 - \frac{36}{m} = 0 \xrightarrow{\times m} 5m^2 - 3m - 36 = 0$

$m = \begin{cases} 3 \\ -2/4 \end{cases}$ → غیر قابل قبول چون $a > 0 \Rightarrow m > 0$ ، $\frac{-b}{2a} = \frac{6}{m} = 2$

۷- $\alpha^2 = \alpha\beta = P = \frac{c}{a} = \frac{2a-1}{1}$ ، $a^2 - 2a + 1 = 0 \Rightarrow a = 1$

۸- $t = x^2 \Rightarrow t^2 - 7t - 5 = 0$ ، $t_1 + t_2 = \frac{-b}{a} = 7$

$t_1 = \pm x_1 , t_2 = \pm x_2$ ، $S = x_1 - x_2 - x_1 + x_2 = 0$

$t^2 - 7t - 5 = 0 \Rightarrow t = \begin{cases} \frac{7+\sqrt{69}}{2} \\ \frac{7-\sqrt{69}}{2} \end{cases}$ $\Rightarrow x^2 = \frac{7+\sqrt{69}}{2} \Rightarrow x = \pm\sqrt{\frac{7+\sqrt{69}}{2}}$

($\frac{7-\sqrt{69}}{2}$) ← چون $x^2 \geq 0$ غیر قابل قبول است

$2P^2 - 3SP - 2S = 2P^2 - 0 = 2\left(-\sqrt{\frac{7+\sqrt{69}}{2}} \cdot \sqrt{\frac{7+\sqrt{69}}{2}}\right)^2 = 2 \cdot \frac{118 + 14\sqrt{69}}{4} = 59 + 7\sqrt{69}$

۹- $y = kx^2 - 4x - 6 \Rightarrow ext \left| \begin{matrix} \frac{2}{k} \\ y \end{matrix} \right.$

$y = k\left(\frac{2}{k}\right)^2 - 4 \cdot \frac{2}{k} - 6 = \frac{4}{k} - \frac{8}{k} - 6 = \frac{-4}{k} - 6$

$y = -4x - 4 \Rightarrow \frac{-4}{k} - 6 = -4\left(\frac{2}{k}\right) - 4 \Rightarrow \frac{4}{k} - 2 = 0 \Rightarrow k = 2$

$\frac{-4}{k} - 6 = \frac{-4}{2} - 6 = -2 - 6 = -8$

۱۰- $-mx^2 + mx + 1 = -m - x \Rightarrow -mx^2 + (m+1)x + 1 + m = 0$

$\Delta < 0 \Rightarrow (m+1)^2 + 4m + 4m^2 = m^2 + 1 + 2m + 4m + 4m^2 = 5m^2 + 6m + 1 < 0$

$\Rightarrow \left(m + \frac{1}{5}\right)(m+1) < 0$ ، ریشه‌ها: -1 ، $-\frac{1}{5}$ ؛ علامت: + − + $\Rightarrow m = \left(-1 , -\frac{1}{5}\right)$

شامل هیچ عدد صحیحی نیست.