نام و نام خانوادگی ....حسین حاجی.... ۱۶/۵ پاسخنامه تشریحی تکلیف شماره ۲.۲ کلاس ..دهم تیزهوش..B.........

۱

$S=\alpha+\beta=5$ $\quad P=\alpha\beta=2$

$\beta^2-5\beta+2=0 \Rightarrow \beta^2=5\beta-2$

$\beta^3=\beta\times\beta^2=\beta(5\beta-2)=5\beta^2-2\beta = 5(5\beta-2)-2\beta = 25\beta-10-2\beta = 23\beta-10$

$\beta^5=\beta^2\times\beta^3=(5\beta-2)(23\beta-10)=115\beta^2-96\beta+20=115(5\beta-2)-96\beta+20$

$=575\beta-230-96\beta+20=479\beta-210$

$\alpha=5-\beta \Rightarrow 4\alpha=20-4\beta \quad 4\alpha+\beta^5=20-4\beta+479\beta-210=475\beta-190$

$5\beta^2=5(5\beta-2)=25\beta-10 \quad \frac{475\beta-190}{25\beta-10}=\frac{95(5\beta-2)}{5(5\beta-2)}=\boxed{19}$ ✓ (۲)

۲

$x_S=\frac{3+(-\frac{1}{5})}{2}=\frac{14}{5}\times\frac{1}{2}=\frac{7}{5}$ $\quad \left(-\frac{b}{2a}\right)$

$-\frac{b}{a}=S=-\frac{b}{2a}\times 2=\frac{7}{5}\times 2=\boxed{\frac{14}{5}}$ (۰/۵)

$A(3,-4)$, $B(-1/5,-4)$ $\xrightarrow{\in f}$ $f(x)=ax^2+bx+c$

$\frac{x_A+x_B}{2}=\frac{x_S}{2} \rightarrow \frac{-b}{2a} \rightarrow \frac{-b}{a}=x_A+x_B=3+(-1/5)=1/5$

۳

$|\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|} \quad \Delta=(2k)^2-4(1)(5)=4k^2-20$

$\sqrt{4k^2-20}=\frac{4k}{3} \quad 4k^2-20=\frac{16k^2}{9} \quad 36k^2-180=16k^2$

$20k^2=180 \Rightarrow k^2=9$

$\frac{k^2}{2}=4/5 \Rightarrow [4/5]=\boxed{4}$ ✓ (۲)

۴

$x=\frac{3-5}{2}=-1 \quad (-1,1) \quad y=a(x+1)^2+1$

$y=a(4)^2+1=16a+1 \quad \frac{\alpha+\beta}{2}=-1 \Rightarrow \alpha+\beta=-2 \quad \alpha^2+\beta^2=5=S^2-2P$

$(-2)^2-2P=4-2P \Rightarrow P=-\frac{1}{2} \quad y=ax^2+2ax+a+1 \quad \alpha\beta=\frac{c}{a}=\frac{a+1}{a}$ (۲)

$\frac{a+1}{a}=-\frac{1}{2} \Rightarrow a=-\frac{2}{3} \quad y=a(0+1)^2+1=a+1 \Rightarrow y=1-\frac{2}{3}=\frac{1}{3} \quad \boxed{(0,\frac{1}{3})}$ ✓

۵

$x_{\text{راس}}=\frac{-b}{2a}=\frac{-5}{-2}=\frac{5}{2} \quad f(\frac{5}{2})=-(\frac{5}{2})^2+5(\frac{5}{2})+m$

$\frac{9}{4}+m>0 \Rightarrow m>-\frac{9}{4} \quad m\geq -2$

تعداد صحیح ← $\boxed{\text{بی‌نهایت}}$ (۰)

$x=\frac{-b\pm\sqrt{\Delta}}{2a} \rightarrow \frac{-5\pm\sqrt{25-4(-1)(m)}}{-2}<\frac{9}{2} \rightarrow \pm\sqrt{25+4m}<4$

$0<25+4m<16 \rightarrow -6/25<m<-2/25 \rightarrow m\in\{-3,-4,-5,-6\} \rightarrow$ ۴ عدد صحیح

۶

$y_{min} = c - \frac{b^2}{4a}$ $\quad 2 = 5m - 1 - \frac{144}{4m}$

$2 = 5m - 1 - \frac{36}{m}$ $\Rightarrow 3 = 5m - \frac{36}{m}$

$5m^2 - 3m - 36 = 0 \Rightarrow m = 3, -\frac{12}{5}$

$m = 3 \Rightarrow m > 0$ ✓ قبول

$x = -\frac{b}{2a} = \frac{12}{6} = 2$ ✓ (۲)

۷

$d^2 = qB$

$qB = 2q - 1 \Rightarrow d^2 = 2q - 1$

$d^2 - 2d + 1 = 0 \Rightarrow (d-1)^2 = 0 \Rightarrow d = 1$ ✓ (۲)

۸

$y = x^2$ $\quad x^4 - 7x^2 - 8 = 0$

$y^2 - 7y - 8 = 0$

$\Delta = 49 + 32 = 81$

$y = \frac{7 \pm \sqrt{81}}{2} \rightarrow y = \frac{7+9}{2} \rightarrow x = \pm\sqrt{\frac{7+9}{2}}$, $P = -\frac{7+9}{2}$, $S = 0$

$4p^2 - 4sp + s^2 = 4a^2 \rightarrow a^2 = (-\frac{7+9}{2})^2 = \frac{81 + 7\sqrt{81}}{2} \Rightarrow 4a^2 = 81 + 7\sqrt{81}$ (۲)

۹

$x_s = \frac{2}{k}$ $\quad y_s = -\frac{4}{k} - 9$ $\quad y = -4x - 4$

$-\frac{4}{k} - 9 = -\frac{8}{k} - 4$ $\quad \frac{4}{k} = 2 \Rightarrow k = 2$

$y_s = -\frac{4}{2} - 9 = -7$ ✓ (۲)

$y = -mx^2 + mx + 1$ $\quad -mx^2 + mx + 1 = -m - x$

$y = -m - x$

$-mx^2 + mx + x + m + 1 > 0$

$-mx^2 + (m+1)x + m + 1 > 0$

$\Delta < 0 \rightarrow (m+1)^2 + 4m(m+1) < 0$

$(m+1)(5m+1) < 0$

$-1 < m < -\frac{1}{5}$

عدد صحیح ✓ (۴)