

$$\alpha^2 - \alpha + 2 = 0 \quad \begin{cases} \alpha^2 = \alpha - 2 \\ \beta^2 = \beta - 2 \end{cases} \quad \beta^2 = \beta(\beta^2) = \beta(\alpha - 2) = \alpha\beta - 2\beta = 2\alpha - 1$$

$$\alpha + \beta = \omega$$

$$\alpha\beta = 2 \Rightarrow \alpha = \frac{2}{\beta} \Rightarrow 2\alpha = \frac{1}{\beta} = 2(\omega - \beta) = 2\omega - 4\beta$$

$$\beta^2 = \beta(\beta^2) = \beta(\alpha\beta)^2 = \beta(2\omega\beta^2 - 2\beta + 2) = 2\omega\beta^2 - 2\beta^2 + 4\beta = 2\omega\beta - 2\omega - 1$$

$$-1 - \beta + 4\beta = 2\omega\beta - 2\omega - 1 \quad \frac{2\omega - 4\beta + 4\omega\beta - 2\omega}{\omega\beta^2} = \frac{4\omega\beta - 4\beta}{\omega\beta^2} = \frac{4\omega}{\omega} = 4$$

$$\alpha_5 = \frac{1}{2} + \frac{(-1/\omega)}{2} = 0.1\omega = -\frac{b}{2a}$$

$$\alpha + \beta = -\frac{b}{a} = 2 \times 0.1\omega = 0.2\omega$$

نظم عمومی پس از این برای یک خطی

$$\alpha^2 + 2k\alpha + \omega = 0 \quad |a - b| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{4k^2 - 4\omega}}{1} = 2\sqrt{k^2 - \omega} = \frac{4}{\omega} k$$

$$\Rightarrow 2\sqrt{k^2 - \omega} = \frac{4}{\omega} k \Rightarrow \frac{2}{\omega} \sqrt{k^2 - \omega} = \frac{2}{\omega} k \Rightarrow \sqrt{k^2 - \omega} = k \Rightarrow k^2 - \omega = k^2 \Rightarrow \omega = 0$$

$$\left[\frac{k^2}{\omega} \right] = \left[\frac{9}{\omega} \right] = \left[\frac{1}{\omega} \right] = 1$$

$$A: (x_0, y) \quad B: (\omega, y) \quad \alpha_5 = \frac{1}{2} + \frac{(-\omega)}{2} = -1 \quad y_5 = 1 \quad \text{پس: } (-1, 1)$$

$$y = a\left(m + \frac{b}{2a}\right)^2 - \frac{a}{4a} \Rightarrow y = a(m+1)^2 + 1$$

$$y = 0 \Rightarrow 0 = a(m+1)^2 + 1 \Rightarrow a(m+1)^2 = -1 \Rightarrow (m+1)^2 = -\frac{1}{a} \Rightarrow \begin{cases} \alpha = -1 + \sqrt{-\frac{1}{a}} \\ \beta = -1 - \sqrt{-\frac{1}{a}} \end{cases}$$

$$\alpha^2 + \beta^2 + \omega = 0 \Rightarrow (\alpha + \beta)^2 - 2\alpha\beta + \omega = 0 \Rightarrow \omega^2 - 2\alpha\beta + \omega = 0 \Rightarrow \omega^2 - 2(-1) + \omega = 0 \Rightarrow \omega^2 + \omega + 2 = 0$$

$$\omega = \frac{-1 \pm \sqrt{1 - 8}}{2} = \frac{-1 \pm \sqrt{-7}}{2} \Rightarrow \sqrt{2\omega + 4} < 1 \Rightarrow \sqrt{2\omega + 4} < 1 \Rightarrow 2\omega + 4 < 1 \Rightarrow 2\omega < -3 \Rightarrow \omega < -1.5$$

$$\Rightarrow -2\omega \leq 4m < -4 \Rightarrow -4m \leq 2\omega < -2 \Rightarrow m \geq \frac{1}{2} \Rightarrow m \in \{-1, -2, -3\}$$

$$y = \frac{1}{2} m^2 - 1 \quad m > 0 \quad y_{min} = -1 \quad a_5 = \frac{-b}{2a} = \frac{1}{2} = \frac{y}{m}$$

$$\Rightarrow -1 = \frac{1}{2} \left(\frac{y}{m} \right)^2 - 1 \quad \Rightarrow \frac{1}{2} \left(\frac{y}{m} \right)^2 = 0 \quad \Rightarrow y = 0$$

$$\Rightarrow \frac{1}{2} m^2 - 1 = 0 \quad \Rightarrow m = \pm \sqrt{2} \quad \Rightarrow m = \sqrt{2} \quad \Rightarrow a_5 = \frac{y}{m} = \frac{0}{\sqrt{2}} = 0$$

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$$a^2 + 2(a+1)a + a - 1 = 0 \quad \Rightarrow a^2 + 2a + a - 1 = 0 \quad \Rightarrow a^2 + 3a - 1 = 0$$

$$a = \frac{-3 \pm \sqrt{9 + 4}}{2} = \frac{-3 \pm \sqrt{13}}{2}$$

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$$a^2 - 2a - 1 = 0 \quad \Rightarrow t^2 - 2t - 1 = 0 \quad \Rightarrow t = \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$$a = \pm \sqrt{t} = \pm \sqrt{1 \pm \sqrt{2}} \quad \Rightarrow s = 0$$

$$P = (1 + \sqrt{2})(1 - \sqrt{2}) = -1 = -\frac{1}{1}$$

$$P^2 - 2SP + S^2 = P^2 = 1 - 2(1 + \sqrt{2})(1 - \sqrt{2}) = 1 - 2(-1) = 3$$

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$$y = Kx^2 - 4x - 4 \quad a_5 = \frac{-b}{2a} = \frac{2}{K}$$

$$y_5 = K \left(\frac{2}{K} \right)^2 - 4 \left(\frac{2}{K} \right) - 4 = \frac{4}{K} - \frac{8}{K} - 4 = -\frac{4}{K} - 4$$

$$y_5 = -\frac{4}{K} - 4 = -1 \quad \Rightarrow -\frac{4}{K} - 4 = -1 \quad \Rightarrow -\frac{4}{K} = 3 \quad \Rightarrow K = -\frac{4}{3}$$

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$$y = -m^2 + m + 1 \quad y = -m - 1 \quad \Rightarrow -m^2 + m + 1 = -m - 1 \quad \Rightarrow -m^2 + 2m + 2 = 0$$

$$\Delta = 4 + 8 = 12 \quad \Rightarrow m = \frac{2 \pm \sqrt{12}}{-2} = \frac{2 \pm 2\sqrt{3}}{-2} = -1 \pm \sqrt{3}$$

$$m \in (-1 - \sqrt{3}, -1 + \sqrt{3})$$

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