تکلیف ۴۴ . آرین بقائی راد ۲۰

(هم سرگروه)

۱)

الف) $2x^2-4x+1 \longrightarrow min \left| \begin{matrix} \frac{-b}{2a} \\ \frac{-\Delta}{4a} \end{matrix} \right. = min \left| \begin{matrix} 1 \\ -1 \end{matrix} \right.$ ✓

ب) $-2x^2+3x-5 \longrightarrow max \left| \begin{matrix} \frac{-b}{2a} \\ \frac{-\Delta}{4a} \end{matrix} \right. = max \left| \begin{matrix} \frac{3}{4} \\ \frac{-31}{8} \end{matrix} \right.$ ✓

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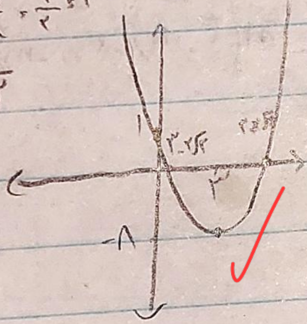
x^2-6x+1 $\quad \frac{-b}{2a} = \frac{6}{2} = 3$

$\frac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$

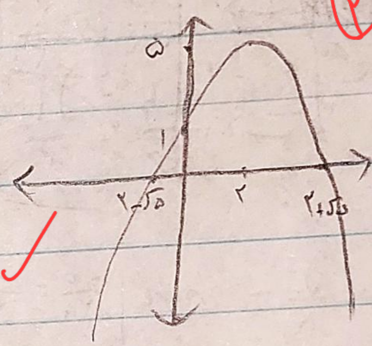


✓

ب) $y = -x^2+4x+1$ $\quad \frac{-b}{2a} = 2 = \frac{-4}{-2}$

$y=5$

$\frac{-4 \pm 2\sqrt{5}}{-2} = 2 \pm \sqrt{5}$



✓

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۳)

$4x^3+kx^2-9x-2 = (x^2-x-2)(4x+m)$

$4x^3+mx^2-4x^2-mx-8x-2m = 4x^3+x^2(m-4)-x(m+8)-2m$

$\left. \begin{matrix} 2m=2 \\ m+8=9 \\ m-4=k \end{matrix} \right\} \Rightarrow m=1 \Rightarrow k=-3$ ✓

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$x^2 - 2mx + m = 0$

$\sqrt{\alpha} - \sqrt{\beta} = 1 \Rightarrow \alpha + \beta - 2\sqrt{\alpha\beta} = 2m - 2\sqrt{m} = 1$

$\sqrt{m}(2\sqrt{m} - 2) = 1 \Rightarrow m = 1 \Rightarrow 2x^2 - x - 1 \to \alpha\beta = \frac{-1}{2}$ ✓ ۲

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$\frac{|\alpha - \beta| \times |m|}{2} = \frac{2}{4} \Rightarrow \left| \frac{\sqrt{(m+2)^2 - 8m} \times m}{4} \right| = \frac{4}{4}$

$\frac{m|m-2|}{4} < \frac{2}{4} \Rightarrow m|m-2| = 2 \begin{cases} m(m-2) = 2 \Rightarrow m^2 - 2m - 2 \begin{cases} m = 2 \\ m = 1 \end{cases} \\ m(2-m) = 2 \Rightarrow -m^2 + 2m - 2 = -4 < 0 \times \end{cases}$ ۲

$y = 2x^2 - 2x + 1 \Rightarrow \frac{-b}{2a} = \frac{2}{2}$

$y = x^2 + x + 1 \Rightarrow \frac{-b}{2a} = \frac{-1}{2}$ ✓

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$ax^2 + 2x + a = \frac{7}{8} \to \min \quad \left| \frac{-b}{2a} = \frac{-2}{2a} \Rightarrow a > 0 \right.$

$\Rightarrow g\left(\frac{9}{4a}\right) - \frac{9}{2a} + a$ ۲

$\frac{9}{4a} - \frac{18}{4a} + a = a - \frac{9}{4a} = \frac{7}{8} \Rightarrow 8a^2 - 18 = 7a$

$\therefore 8a^2 - 7a - 18 \begin{cases} a = 2 \checkmark \\ a = \frac{-9}{8} \times \end{cases}$ ✓ $\Rightarrow a = 2$ یک مقدار

$\alpha\ a = -1 \longleftarrow (a+1)(a-3) \longrightarrow a = 3$ ✓

$P_{\text{ضرب}} > 0 \Rightarrow a > 0$

۷)

$x^2 - (a+1)x + a = 0 \Rightarrow \frac{\sqrt{\Delta}}{1} = 2 \Rightarrow a^2 + 2a + 1 - 4a = 4 \Rightarrow a^2 - 2a + 1 = 4 = a^2 - 2a - 3 = 0 \Rightarrow a = 3$

$x^2 - (2a+1)x + b = x^2 - 10x + b = 0 \rightarrow \frac{\sqrt{\Delta}}{1} = 2 \Rightarrow 100 - 4b = 4 \Rightarrow 4b = 96$

$P_{\text{ضرب}} = 3$

$b = 24 = P_{\text{ضرب}}$

$24 - 3 = 21$

(۲) ✓

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۸)

$y = -ax^2 + ax + 2 \rightarrow \left| \frac{a}{2a} = \frac{1}{2} \Rightarrow 2 + \frac{1}{4}a = -1 \right.$

$\Rightarrow -\frac{1}{4}a + \frac{1}{2}a + 2 = 2 + \frac{1}{4}a \Rightarrow 2 + \frac{1}{4}a = ~~\frac{1}{4}b - \frac{1}{2}b~~ - 1$

$2 + \frac{1}{4}a = -1 \Rightarrow -3 = \frac{1}{4}a \Rightarrow a = -12$

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$y = 2bx^2 - bx - 1 \rightarrow \left| \frac{b}{4b} = \frac{1}{4} \right.$

$\Rightarrow \frac{1}{8}b - \frac{2}{8}b - 1 = \frac{-1}{8}b - 1 \Rightarrow \left(\frac{1}{4} \times 12\right) - (3) + 2 = -\frac{4}{4} = \frac{-1}{8}b - 1$ ✓

-1

$\frac{3}{4} = \frac{-1}{8}b \Rightarrow b = -6$

$\Rightarrow b - a = -6 + 12 = 6$

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۹)

$\frac{\beta}{25\alpha} = \alpha\beta \Rightarrow 25\alpha^2\beta = \beta \Rightarrow 25\alpha^2 = 1$

$\Rightarrow \alpha^2 = \frac{1}{25} \Rightarrow \alpha = \pm\frac{1}{5}$ $\left| \Rightarrow \begin{matrix} \beta = 1 \\ \alpha = \frac{-1}{5} \end{matrix} \right\} \Rightarrow -5x^2 + 4x + b$ ✓

ناحیه اول

$\frac{-4}{25\alpha} \begin{cases} \rightarrow \frac{-4}{5} = \frac{1}{5} + \beta \Rightarrow \beta = -1 \rightarrow \beta < \alpha$ ✗ $\quad \alpha = \frac{1}{5} \\ \rightarrow \frac{4}{5} = \frac{-1}{5} + \beta \Rightarrow \beta = 1$ ✓ $\quad \alpha = \frac{-1}{5} \quad \beta > \alpha \end{cases}$

$\Rightarrow \alpha = \frac{-1}{5} \Big\} \Rightarrow \frac{-b}{2a} = \frac{-4}{5\alpha} = \frac{-4}{-1} = 0/4 = x$

$\Rightarrow y = \frac{9}{5}$

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$$\frac{a+b-1}{1} = ab \Rightarrow a+b \leq ab+1$$

$$a^2+b^2-12 = a+b \quad \Rightarrow P, S-1$$

$$a^2+b^2-12 < ab+1, \quad S>0$$

$$\Rightarrow S^2-2P-12 < P+1$$

$$\Rightarrow S^2-2S+2-12 < S \Rightarrow S^2-3S-10 < 0 \Rightarrow (S-5)(S+2) \Rightarrow S \leq 5, a+b$$