امیدی‌پوش ۲۰

۱) الف) $a>0 \to$ min, ext $\left| \begin{array}{l} -\frac{b}{2a} \to -\frac{(-2)}{2(2)} = 1 \\ 2(1)^2 - 4(1) + 1 = -1 \end{array} \right.$

ب) $a<0 \to$ max, ext $\left| \begin{array}{l} -\frac{3}{2(-2)} = \frac{3}{4} \\ -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) - 5 = -\frac{41}{8} \end{array} \right.$

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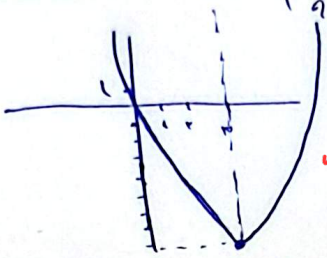
الف) $y = x^2 - 4x + 1 \xrightarrow{a>0} \min$, ext $\left| \begin{array}{l} -\frac{(-4)}{2} = 2 \\ 4 - 8 + 1 = -3 \end{array} \right.$

x	2	3	4	5
y	-3	-3	1	6

محور تقارن



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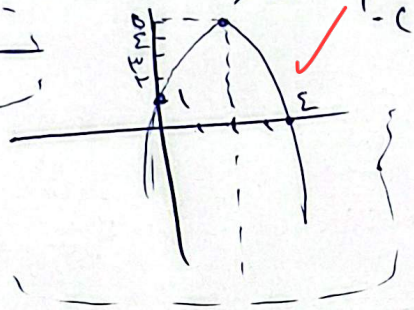
ب) $y = -x^2 + 4x + 1 \xrightarrow{a<0}$ max, ext $\left| \begin{array}{l} -\frac{4}{2(-1)} = 2 \\ -(4) + 8 + 1 = 5 \end{array} \right.$

x	1	2	3	4	5
y	4	5	4	1	

محور تقارن



$\alpha\beta = -2$

$\alpha + \beta = 1 \to \alpha = 1 - \beta$

$(1-\beta)(\beta) = -2 \to -\beta^2 + \beta = -2 \to \beta^2 - \beta - 2 = 0 \begin{cases} \beta = 2 \\ \beta = -1 \end{cases}$

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$4x^3 + 12x^2 - 9x - 2 = 0 \begin{cases} x = 2 \to 4(2)^3 + 12(2)^2 - 9(2) - 2 = 0 \to 4k = -12 = -k = 3 \\ x = -1 \to 4(-1)^3 + 12(-1)^2 - 9(-1) - 2 = 0 \to k = -3 \end{cases}$

$k = -3$

$|\sqrt{\alpha} - \sqrt{\beta}| = 1 \to \alpha + \beta - 2\sqrt{\alpha\beta} = 1 \Big\} x^2 - 3mx + m = 0 \to$

$S = 3m$ و $P = m$

$3m - 2\sqrt{m} - 1 = 0 \xrightarrow{a+b+c=0}$

$\begin{cases} ① = 1 \to \sqrt{m} = 1 \to m = 1 \\ ② \frac{c}{a} = -\frac{1}{3} \to \sqrt{m} = -\frac{1}{3} \to \text{ق ق} \end{cases}$

$\to 2x^2 - mx - m = 0 \to P = \alpha\beta = -\frac{m}{2} = -\frac{1}{2}$

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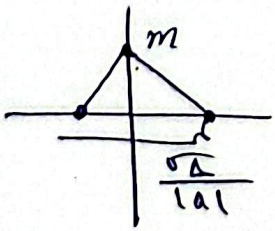
$S_{\text{مثلث}} = \frac{1}{2} \times \frac{\sqrt{\Delta}}{|a|} \times m = \left|\frac{3}{4}\right| \to \frac{1}{2} \times m \times \frac{\sqrt{m^2 + 4 - 4m}}{2} = \left|\frac{3}{4}\right|$



$\sqrt{m^2 - 4m + 4} = \sqrt{(m-2)^2} \to m|m-2| = |3|$

$m|m-2| \begin{cases} -3 \to m \leq 2 \to -m^2 + 2m + 3 = 0 \to m = -1 \quad m \leq 2 \\ 3 \to m \geq 2 \to m^2 - 2m - 3 = 0 \to m = 3 \quad m = -1 \quad m \geq 2 \end{cases}$

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$m = 3 \to y = x^2 + 3x + 2 \to x = -\frac{b}{2a} = +\frac{3}{2}$

$m = -1 \to y = x^2 - x + 2 \to -\frac{b}{2a} = -\frac{1}{2}$

$\min \to a>0 \Big\} \; y_s = \frac{-b}{4a} = \frac{-9-4(a^2)}{4a} = \frac{7}{8} \to 32a^2-28a-28=0 \to$

$8a^2-7a-14=0 \to a^2-7a-14=4 \Rightarrow (a+a)(a-14)=0$

$\begin{cases} -\frac{a}{8} \text{ قابل قبول نیست} \\ \frac{14}{8}=2 \to P=\{2\} \end{cases}$ $a>0$ ✓

$x^2-(a+1)x+a=0 \to \frac{\sqrt{\Delta}}{|a|}=2 \to \sqrt{a^2+1+2a-4a}=2$

$(a-1)^2$

$|a-1|=2 \xrightarrow{a>1} a=3 \to x^3-4x+3=0 \to (x-1)(x-3)=0 \to P_1=3$

$\xrightarrow{a<1} a=-1 \to x^2-1=0 \Rightarrow x=\pm 1$ (طبیعی نیست)

$x^2-(3a+1)x+b=0 \to \frac{\sqrt{\Delta}}{|a|}=2 \to \sqrt{9a^2+1+2a-4b}=2 \xrightarrow{x=3}$

$\sqrt{81+1+18-4b}=2 \to 100-4b=4 \to b=24 \to P_2=24$

$|P_2-P_1|=24-3=21$ ✓

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① $y=-ax^2+ax+2 \to ext \Big| \begin{matrix} \frac{b}{2} \\ -\frac{a}{4}+\frac{a}{2}+2=\frac{a+8}{4} \end{matrix}$

② $y=2bx^2-bx-1 \to ext \Big| \begin{matrix} \frac{1}{4} \\ -\frac{b-8}{8} \end{matrix}$

③ $x=\frac{1}{2}, y=\frac{a+8}{4} \to \frac{b}{2}-\frac{b}{2}-1=\frac{a+8}{4} \to a=-12 \Big\} x=\frac{1}{4}, y=\frac{-b-8}{8} \to$

$\frac{12-48+32}{16}=\frac{-2b-16}{16} \to -4=-2b-16 \to b=-6$

$b-a=-6-(-12)=6$ ✓

$y = 2ax^2 + 4x + \beta$ $\begin{cases} (1)\ 2a^3 + 4\alpha + \beta \to 2\alpha^3 + 4\alpha - 5\alpha = 0 \xrightarrow{\star} \\ (2)\ 2\beta^2\alpha + 5\beta = 0 \to 5\beta(2\alpha\beta + 1) = 0 \xrightarrow{\Delta} \end{cases}$

$\xrightarrow{\star} a(2a^2 - 5) = 0 \to a(5a-1)(5a+1) \begin{cases} 0 \\ \frac{1}{5} \to \beta = -1 \to \alpha, \beta \text{ غیرقابل} \\ -\frac{1}{5} \to \beta = 1 \checkmark \end{cases}$

$\xrightarrow{\Delta} \begin{cases} \beta = 0 \\ 5\alpha\beta + 1 = 0 \to 2\beta = -\frac{1}{5} = \frac{\beta}{2\alpha} \to \beta = -5\alpha \end{cases}$

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$\alpha = -\frac{1}{5}, \beta = 1 \to -5x^2 + 4x + 1 \to ea\ t \left| \begin{array}{l} -\frac{4}{2(-5)} = \frac{2}{5} \checkmark \\ -5(\frac{2}{5})^2 + 4(\frac{2}{5}) + 1 = \frac{9}{5} \end{array} \right.$ تساوی

$a + b = s,\ ab = p \mid s = -(-(a^2 + b^2 - 12)) = s^2 - 2p - 12$
$p = a + b - 1 = s - 1$ $\xrightarrow{\star}$

$\xrightarrow{\star} s^2 - 2(s-1) - 12 = s \to s^2 - 2s - 10 = s \to s^2 - 3s - 10 = 0 \to s \begin{cases} 5 = a + b \to \\ -2 = a + b \to \end{cases}$

$\longrightarrow p = 5 - 1 = 4$

$\longrightarrow p = a + b - 1 = -2 - 1 = -3$ چون ریشه‌ها طبیعی هستند نمی‌تواند منفی بشود

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