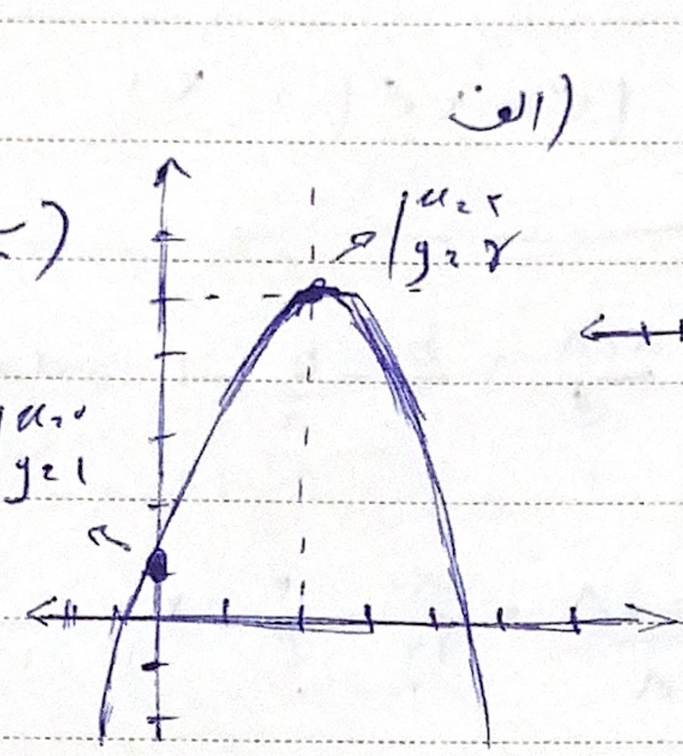
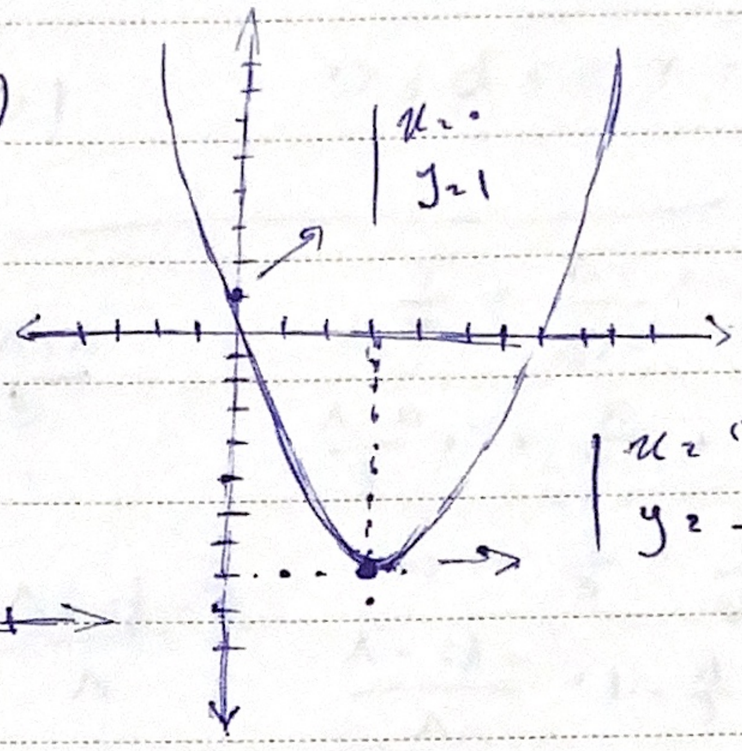
۱- ب) $\max ext \left| \begin{array}{l} x = \frac{3}{4} \\ y = -2\alpha\frac{9}{16} + 3.\frac{3}{4} - 2 = \frac{-21}{8} \end{array} \right.$

الف) $\min ext \left| \begin{array}{l} x = \frac{-b}{2a} = \frac{4}{4} = 1 \\ y = 2 - 4 + 1 = -1 \end{array} \right.$

۲- الف)



ب)



$\max ext \left| \begin{array}{l} x = 2 \\ y = 2 \end{array} \right.$ $\min ext \left| \begin{array}{l} x = 3 \\ y = -8 \end{array} \right.$

۳- معادله درجه سوم در اینجا یک ریشه مضاعف دارد،

$4x^3 + kx^2 - 9x - 2 = 0 \rightarrow (x-\alpha)(x-\beta)^2 = 0$

$\Rightarrow (x^2 - (\alpha+\beta)x + \alpha\beta)(x-\beta) = 0 \Rightarrow (x^2 - x - 2)(x-\beta) = 0 \Rightarrow (x-2)(x+1)(x-\beta) = 0$

عبارت بر $(x+1)$ بخش پذیر است $\Rightarrow 4 - 9 = k - 2 \Rightarrow k = -3$

$\Rightarrow \beta = 2, \alpha = -1$

۴- $\sqrt{\alpha} - \sqrt{\beta} = 1 \Rightarrow \alpha + \beta - 2\sqrt{\alpha\beta} = 1 \Rightarrow 5m - 2\sqrt{m} = 1 \Rightarrow 5t^2 - 2t - 1 = 0$

$\Rightarrow t = 1, \left(-\frac{1}{5}\right) \rightarrow$ غیرقابل قبول $\Rightarrow \sqrt{m} = 1 \Rightarrow m = 1$

$\Rightarrow \frac{c}{a} = \alpha\beta = 0.1 \Rightarrow 2x^2 - x - 1 = 0 \Rightarrow 2x^2 - mx - m = 0$

۵- $\frac{\frac{\sqrt{\Delta}}{|a|}}{2} \times m = \frac{c}{4} \Rightarrow \frac{\left(\sqrt{(m-2)^2}\right)m}{4} = \frac{5}{4} \Rightarrow m^2 - 2m - 5 = 0 \Rightarrow m = 1 - \sqrt{6}$

$\Rightarrow y = \begin{cases} x^2 - 2x + 1 \\ x^2 + x + 1 \end{cases} \Rightarrow x = \frac{-b}{2a} = \frac{3}{4}, -\frac{1}{2}$

۶- $y = ax^2 + 2x + a$ $\min$ $\Rightarrow a > 0$ $y = \frac{-\Delta}{4a} = \frac{4a^2 - 9}{16a} = \frac{5}{8}$

$\Rightarrow 8a^2 - 5a - 18 = 0 \Rightarrow a = \frac{5 \pm \sqrt{484}}{16} = 2, -\frac{9}{8} \rightarrow$ غیرقابل قبول

$\Rightarrow$ به ازای ۱ مقدار

۷:

$x^2-(a+1)x+a=0 \Rightarrow |\alpha-\beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2+1+2a-4a}}{1} = 2$

$\Rightarrow |a-1| = 2 \Rightarrow a = -1, 3$ (-1 → غیر قابل قبول) $\Rightarrow x^2-10x+b=0 \Rightarrow |\alpha-\beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{100-4b}}{1} = 2$

$\Rightarrow 2\sqrt{25-b} = 2 \Rightarrow b = 24$

$|3-24| = \underline{21}$

۸ـ

$ext \left| \begin{array}{l} x = \frac{-b}{2a} = \frac{-a}{-2a} = \frac{1}{2} \\ y = -\frac{a}{4} + \frac{a}{2} + 2 = \frac{a+8}{4} \end{array} \right.$

$\frac{a+8}{4} = \frac{b}{2} - \frac{b}{2} - 1 \Rightarrow a+8 = -4 \Rightarrow \underline{a = -12}$

$ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4} \to x \\ y = \frac{b}{8} - \frac{b}{4} - 1 = \frac{-b-8}{8} \end{array} \right.$

$\frac{-b-8}{8} = \frac{12}{16} - \frac{12}{4} + 2 = \frac{-4}{16} \Rightarrow -b-8 = -2 \Rightarrow \underline{b = -6}$

$\Rightarrow b - a = -6(-12) = \underline{6}$

۹ـ

$y = 25ax^2 + 4x + \beta \Rightarrow \alpha\beta = \frac{\beta}{25\alpha} \Rightarrow 25\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{5}$

✗ اگر $\alpha = \frac{1}{5} \to 25 \cdot \frac{1}{5} \cdot \frac{1}{25} + 4 \cdot \frac{1}{5} + \beta = 0 \Rightarrow \beta = -1 \to$ غیرقابل قبول

✓ اگر $\underline{\alpha = \frac{-1}{5}} \to 25 \cdot \frac{-1}{5} \cdot \frac{1}{25} + 4 \cdot \frac{-1}{5} + \beta = 0 \Rightarrow \underline{\beta = 1}$

$\Rightarrow y = -5x^2 + 4x + 1 \quad ext \left| \begin{array}{l} x = \frac{-b}{2a} = \frac{2}{5} \\ y = -5 \cdot \frac{4}{25} + \frac{8}{5} + 1 = \frac{9}{5} \end{array} \right. \Rightarrow$ ناحیه اول

۱۰ـ $1 =$ یکی از اعداد $\Rightarrow$ حاصل ضرب ۲ عدد طبیعی $<$ جمع آن؟

$\Rightarrow a+b = a^2+b^2-12 \Rightarrow 1+b = 1+b^2-12 \Rightarrow b^2-b-12=0$

$\Rightarrow b = \underline{4}, -3 \longrightarrow$ غیر قابل قبول $\Rightarrow a+b = \underline{1+4 = 5}$