نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۰۴ .... کلاس .....

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الف) $y = 2x^2 - 4x + 1$ ، $y > 0$ ← دارد min ، ext $\begin{vmatrix} 1 \\ -1 \end{vmatrix}$

ب) $y = -2x^2 + 3x - 5$ ، $y < 0$ ← دارد max ، ext $\begin{vmatrix} \frac{3}{4} \\ -\frac{31}{8} \end{vmatrix}$

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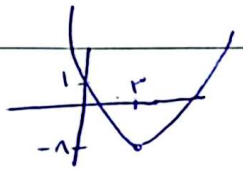
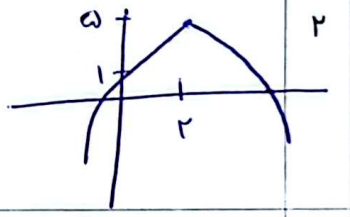
الف) $y = x^2 - 4x + 1$ ، min $\begin{vmatrix} 3 \\ -8 \end{vmatrix}$



ب) $y = x^2 + 4x + 1$ ، max $\begin{vmatrix} 2 \\ 5 \end{vmatrix}$



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$4x^3 + 4x^2 - 9x - 2 = 0$ ، α, β, γ ریشه

$\alpha\beta = -2$ ، $\alpha + \beta = 1$

$x^2 - Sx + P$ ، $\alpha + \beta = 1$ ، $\frac{S}{P}$ ، $\alpha\beta = -2$ ← $\alpha^2 - \alpha - 2 = 0$

$\alpha = -1, \beta = 2$ ، $\alpha = 2, \beta = -1$

if $\alpha = -1$ ، $\beta = -1$ ← $4x^3 + kx^2 - 9x - 2 = 0$ ← $-4 + k + 9 - 2 = 0 \Rightarrow k = -3$

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$\sqrt{\alpha} - \sqrt{\beta} = 1$ (^{2}) ← $\alpha + \beta - 2\sqrt{\alpha\beta} = 1$ ← $S - 2\sqrt{P} = 1$

$3m - 2\sqrt{m} = 1$

← $3m - 2\sqrt{m} - 1 = 0$ ← $m = 1$ ← $2m^2 - 9m - 1 = 0$ ← $P = \frac{c}{a} = \boxed{-\frac{1}{2}}$

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$y = 2x^2 - (m+2)x + m$ ، $\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{m^2 - 4m + 4}}{2} = \frac{|m-2|}{2}$

$5 = \frac{1}{2} \times \frac{|m-2|}{2} m = 3 \Rightarrow |m-2| m = 3 \Rightarrow m^2 - 2m - 3 = 0$ ← $m = 3$ ، $m = -1$

$y = x^2 - mx + 1$

$y = x^2 - 3x + 1$ ← $x_S = \frac{-b}{2a} = \boxed{\frac{3}{2}}$

$y = x^2 + x + 1$ ← $x_S = \frac{-b}{2a} = \boxed{-\frac{1}{2}}$

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$y = ax^2 + 3x + a \xrightarrow{x_1 = \frac{-3}{2a}} a\left(\frac{-3}{2a}\right)^2 + 3\left(\frac{-3}{2a}\right) + a = \frac{-9 + 4a^2}{4a} = \frac{7}{4}$

$\Rightarrow 8a^2 - 7a - 18 = 0 \Rightarrow a = \frac{7 \pm 25}{16}$

$a = 2$ ✓

$a = \frac{-18}{16} = \frac{-9}{8}$ غ ق، $a > 0$

یک مقدار a

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$ax^2 - (a+1)x + a = 0 \xrightarrow[\text{متوالی طبیعی}]{a+b+c=0}$ ریشه‌ها $x_1 = 1$ $x_2 = 3$

$x_1 x_2 = \frac{a}{1} = 3 \Rightarrow a = 3$

$m^2 - (3a+1)m + 6 = 0 \xrightarrow{\text{ضرب}} m^2 - 10m + 6 = 0$ ضرب $= 6$

$\begin{cases} m_1 = 4 \\ m_2 = 6 \end{cases}$

$P_2 - P_1 = 24 - 3 = 21$

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$y = -ax^2 + ax + 2 \xrightarrow{\text{ent}} \left|\begin{matrix} \frac{1}{2} \\ \frac{a+1}{4} \end{matrix}\right.$ $x = \frac{1}{4}\ y = \frac{-b-1}{8}$

$\frac{-1}{4} + 2 \times \frac{3}{4} = \frac{-b-1}{8} \Rightarrow b = -4$

$y = 2bx^2 - bx - 1 \xrightarrow{\text{ent}} \left|\begin{matrix} \frac{1}{4} \\ \frac{-b-1}{8} \end{matrix}\right.$ $x = \frac{1}{4}\ y = \frac{a}{4} + 2$

$\frac{a}{4} + 2 = \frac{b}{2} \cdot \frac{1}{16} - 1 \Rightarrow a = -12$

$b - a = -4 - (-12) = 8$

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α, β ریشه‌های β $y = 2\alpha x^2 + 4x + \beta \rightarrow \alpha + \beta = \frac{-4}{2\alpha}$

$\alpha\beta = \frac{\beta}{2\alpha} \rightarrow 2\alpha^2 = 1 \Rightarrow \alpha = \pm\frac{1}{\sqrt{2}}$

$\alpha = \frac{1}{\sqrt{2}} \rightarrow \beta = -1$ ✗

$\alpha = \frac{-1}{\sqrt{2}} \rightarrow \beta = 1$ ✓

$x_s = \frac{-b}{2a} = \frac{-4}{-2\sqrt{2}} = \frac{2}{\sqrt{2}}$

$y = -\sqrt{2} \times \frac{4}{2\sqrt{2}} + \frac{1}{\sqrt{2}} + 1 = \frac{9}{\sqrt{2}}$ — راس و نابه اول

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$y = x^2 - (a^2 + b^2 - 12)x + a + b - 1 = 0$

$\begin{cases} a + b = a^2 + b^2 - 12 = S \\ ab = a + b - 1 = S - 1 = a^2 + b^2 - 13 = P \end{cases}$

$S = S^2 - 2P - 12 \rightarrow S = S^2 - 2S + 2 - 12 \rightarrow$

$S^2 - 3S - 10 = 0$

$(S-5)(S+2) = 0$

$\rightarrow S = 5$ ✓

$\rightarrow S = -2$ غ ق