الف) $y = 2x^2 - 4x + 1$

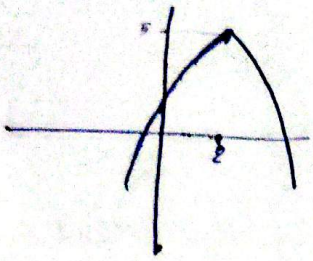
min $\left| \begin{array}{l} -\frac{b}{2a} = 1 \\ -\frac{\Delta}{4a} = \frac{-(16-8)}{4\times 2} = -1 \end{array} \right.$

ج) $y = -2x^2 + 3x - 5$

max $\left| \begin{array}{l} -\frac{b}{2a} = \frac{3}{4} \\ -\frac{\Delta}{4a} = \frac{-(9-40)}{-8} = \frac{31}{8} \end{array} \right.$

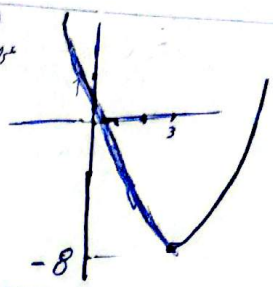
ب) $y = -x^2 + 4x + 1$

ext $\left| \begin{array}{l} -\frac{b}{2a} = -\frac{4}{-2} = 2 \\ -\frac{\Delta}{4a} = -\frac{20}{-4} = 5 \end{array} \right.$



د) $y = x^2 - 6x + 9$ دو ناصفر

ext $\left| \begin{array}{l} \frac{b}{2a} = 3 \\ -8 \end{array} \right.$



$4x^3 + kx^2 - 9x - 2 = 0$ ریشه‌ها α, β

$\alpha + \beta = 1$

$\alpha\beta = -2 \Rightarrow \alpha = -\frac{2}{\beta} \Rightarrow \frac{-2+\beta^2}{\beta} = 1 \Rightarrow \beta^2 - \beta - 2 = 0$ (2)

β, α ← (1)

$\alpha \Rightarrow 4x^3 + kx^2 - 9x - 2 = 0$

$32 + 4k - 18 - 2 = 0 \Rightarrow 4k + 12 = 0 \Rightarrow 4k = -12 \Rightarrow \boxed{k = -3}$

$\beta \Rightarrow 4x^3 + kx^2 - 9x - 2 = 0$

$-4 + k + 9 - 2 = 0 \Rightarrow k + 3 = 0 \Rightarrow \boxed{k = -3}$

$x^2 - 3mx + m = 0$ $\sqrt{\alpha} - \sqrt{\beta} = 1 \Rightarrow \alpha + \beta - 2\sqrt{\alpha\beta} = 1 \Rightarrow \frac{\alpha+\beta}{3m} + 2\frac{\sqrt{\alpha\beta}}{m} - 1 = 0$

$\Rightarrow t^2 - 2t - 1 = 0 \left\langle \begin{array}{l} t = -\frac{1}{3}\alpha \\ t = 1 \end{array} \right.$

$t = 1 \rightarrow \sqrt{m} = 1 \Rightarrow m = 1$

$P_2 = \frac{c}{a} = -\frac{m}{2} = \boxed{-\frac{1}{2}}$

$y = 2x^2 - (m+2)x + m$ ریشه‌ها r_1, r_2 $b = m$ زوج‌اند

$|r_2 - r_1| = \frac{\sqrt{\Delta}}{|a|} \Rightarrow \Delta = 8m - (2+m)^2 = m^2 + 4m + 4 + 8m$

$S = \frac{1}{2} \times |m| \times \frac{|m-2|}{2} \Rightarrow \frac{|m||m-2|}{4} = \frac{3}{4} \Rightarrow |m||m-2| = 3$

$|m(m-2)| = 3 \Rightarrow \pm 3 = m(m-2) \Rightarrow m^2 - 2m - 3 = 0$

$(m-3)(m+1) = 0$

$m = 3 \quad \checkmark \qquad m = -1 \quad \checkmark$

$y = 2x^2 - mx + 1 \Rightarrow x_{\text{رأس}} = \frac{m}{2} \Rightarrow y = (\frac{m}{2})^3 - m(\frac{m}{2}) + 1$

$= \frac{m^2}{4} - \frac{m^2}{2} + 1 = 0 \Rightarrow 1 - \frac{m^2}{4}$

if $m = 3 \Rightarrow y = 1 - \frac{9}{4} = -\frac{5}{4}$

if $m = -1 \Rightarrow y = 1 - \frac{1}{4} = \frac{3}{4}$

$a>0$ ⇒ min ... -6

$x_3 \Big| -\frac{D}{4a} = -\frac{3}{2a}$

$a\left(-\frac{3}{2a}\right)^2 + 3\left(-\frac{3}{2a}\right) + a \Rightarrow a + \frac{9}{2a} - \frac{9}{4a^2}a = \frac{9}{4}a - \frac{18}{4a} + a \Rightarrow a - \frac{9}{4a}$

$\frac{8}{7} = \frac{9}{4a} - a \Rightarrow 7a = 18 - 8a^2 \quad 0 = 18 - 7a - 8a^2$

$\Delta = (-7)^2 - (4)(8)(-18) = 625 \Rightarrow \sqrt{625} = 25 \Rightarrow a = \frac{7 \pm 25}{16}$ ⟨ $a=2$ ✓ ; $a = -\frac{9}{8}$ ✗

$x^2 + ax - x(a+7) = 0$ ⟨ α ; $\beta \Rightarrow \beta = 2\alpha + 2$ ... $x^2 - x(1+3a) + b = 0$ ⟨ z -7

$\alpha + \beta = a + 7 \rightarrow 2\alpha + 2 = a + 7 \Rightarrow a = 2\alpha + 1 \Rightarrow \alpha\beta = 2\alpha + 1 \Rightarrow \alpha = \frac{1}{\beta - 2}$

$xz = b$ ⟶ $x = 4$, $z = 6$ ⟶ $b = 24$, $a = 3$

$xz = b$, $x + z = 9 + 1 = 10$

$x + z = 3a + 1$

$24 - 3 \neq 27$

$y = -ax^2 + ax + 2 \rightarrow$ ext $\Big| -\frac{b}{2a} = \frac{1}{2}$, $\frac{\Delta}{4a} = \frac{a^2 + 8a}{4a}$ -8

$y = 2bx^2 - bx - 1 \rightarrow$ ext $\Big| -\frac{b}{2a} = \frac{1}{4}$, $-\frac{\Delta}{4a} = \frac{b^2 + 8b}{8b} \Rightarrow 2bx^2 - bx - 1 = 0 \Rightarrow x = \frac{1}{2} \Rightarrow \frac{1}{2}b - \frac{1}{2}b - 1 = \frac{a^2 - 8a}{4a}$

$\Rightarrow a^2 + 8a = -4a \Rightarrow a^2 + 12a = 0$ ⟨ 0 ✗ ; -12 ✓

$-ax^2 + ax + 2 \Rightarrow 12x^2 - 12x + 2 \Rightarrow x = \frac{1}{4} \Rightarrow -\frac{1}{4} = \frac{b^2 + 8b}{8b} \Rightarrow -8b = 4b^2 + 32b$

$\Rightarrow 4b^2 + 40b = 0 \Rightarrow 4b(b + 10) = 0 \Rightarrow b = -10$ ∎ $-10 - (-12) = 2$

$\alpha + \beta = -\frac{4}{25a}$, $\alpha\beta = \frac{b}{25a} \Rightarrow 25a^2 + 25ab$, $25a^2 b = b$

$25a^2 = 1 \Rightarrow a = \pm\frac{1}{5} \Rightarrow 25ab = -5 \Rightarrow 25 \cdot \frac{1}{5} \cdot b \Rightarrow b = -1$

$25ab = -5$, $25 \cdot \left(\frac{-1}{5}\right) \cdot b \Rightarrow b = 1$ ⟨ $b > a \Rightarrow b = -1$ ✗ ; $b = 1$ ✓ -9

$\Rightarrow \alpha = -\frac{1}{5}$, $\beta = 1$ ∎

$x^2 - (a^2 + b^2 - 12)x + (a + b - 1) = 0 \Rightarrow S \Rightarrow a^2 + b^2 - 12 = a + b$ (1)

$P \Rightarrow a + b - 1 = ab$ (2)

$a^2 + b^2 = (a+b)^2 - 2ab \Rightarrow$ (1)(2) $(\underbrace{a+b}_{y})^2 - 2(\underbrace{a+b}_{y} - 1) - 12 = a + b$ -10

$y^2 - 3y - 10 = 0 \Rightarrow (y - 5)(y + 3) = 0$ ⟨ $a + b = 5$; $a + b = -3$