امیدی روشن 14,5

الف) $y = 3x^2 - 2x \xrightarrow[\min]{a>0} ext \left| \begin{array}{l} \frac{-(-2)}{2\times 3} = \frac{1}{3} \\ 3\times\frac{1}{9} - \frac{2}{3} = -\frac{1}{3} \end{array} \right.$

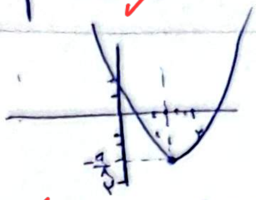
از ناحیه ۳ نمی‌گذرد

2

ب) $y = -x^2 + 4x \xrightarrow[\max]{a<0} ext \left| \begin{array}{l} \frac{-4}{2\times(-1)} = 2 \\ -4 + 8 = 4 \end{array} \right.$

از ناحیه ۲ نمی‌گذرد

الف) $y = 2x^2 - 5x + 2 \xrightarrow[\min]{\Delta>0} ext \left| \begin{array}{l} \frac{-(-5)}{2\times 2} = \frac{5}{4} \\ 2\times\frac{25}{16} - \frac{25}{4} + 2 = -\frac{9}{8} \end{array} \right.$



از نواحی اول و دوم و چهارم

ب) $y = -x^2 + 4x - 1 \xrightarrow[\max]{a<0} ext \left| \begin{array}{l} \frac{-4}{2\times(-1)} = 2 \\ -4 + 8 - 1 = 3 \end{array} \right.$

2

از نواحی اول سوم و چهارم

$x^2 - x - 3 = 0 \longrightarrow \underbrace{\alpha, \beta}_{\text{ریشه}} \to \begin{array}{l} \alpha + \beta = 1 \\ \alpha\beta = -3 \end{array} \quad \alpha - \beta = \frac{\sqrt{\Delta}}{|a|} \to \frac{\sqrt{1 - 4(-3\times 1)}}{1} = \sqrt{13}$

1,5

الف) $\frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

ب) $S^2 - 2P \to 1 - 2(-3) = 7$

ج) $S^3 - 3SP \to 1 - 3(-3\times 1) = 10$

د) $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta) = \sqrt{13}\,(7 - 3) = 4\sqrt{13}$

د) $\alpha^3 - \beta^3 \to (\alpha - \beta)^3 = \alpha^3 - \alpha^2\beta + \alpha\beta^2 - \beta^3 \to \alpha^3 - \beta^3 - \alpha^2\beta + \alpha\beta^2$

$\underbrace{\alpha^3 - \beta^3}_{} - \underbrace{\alpha\beta}_{P}\underbrace{(\alpha - \beta)}_{\frac{\Delta}{|a|} = \sqrt{13}} \to S^3 - P\frac{\Delta}{|a|}$

$1 - (-3\times\sqrt{13})$

$1 + 3\sqrt{13}$

این معادله طبق (داتی) اول یک جواب دارد پس تنها نقطه برخورد حصاری است

در نتیجه معادله دوم نباید جواب داشته باشد پس نباید ریشه مضاعف دو که خارج $\Delta = 0$

$x^2 - ax + a \to \underset{\text{نباید جواب داشت}}{\Delta < 0}$

$a^2 - 4a < 0$ مجموعه مقادیر a = $(0, 4)$

2

$+ \quad 0 \quad - \quad 4 \quad +$

$3x^2 - 12x - a = 0 \quad \alpha, \beta$ ریشه، $2\alpha^2 + \beta^2 - 4a = 7 \,,\; \alpha + \beta = 4 \quad \alpha\beta = -\frac{a}{3}$

$\left.\begin{array}{l} \alpha^2 + \beta^2 + \alpha^2 - 4\alpha = 7 \\ 3\alpha^2 - 12\alpha = a \to \alpha^2 - 4\alpha = \frac{a}{3} \end{array}\right\} 16 + \frac{2a}{3} + \frac{a}{3} = 7 \to 16 + a = 7 \to a = -9 \to \alpha\beta = 3$

$3x^2 - 12x + 9 = 0 \xrightarrow{\div 3} x^2 - 4x + 3 = 0 \quad (x - 1)(x - 3)$

$\to 3$ — ریشه بزرگ‌تر $\quad \alpha\beta = -\frac{a}{3} = -3$

2

$A(2a+3, a-2), B(7-2a, a-2)$ — متقارن، متمایز، S کامل — $S(b, b-2)$

$b = \frac{2a+3+7-2a}{2} = 5 \to S(5, 3)$ ۲

$y = \alpha(x-5)^2 + 3 \to 1 = \alpha(-4)^2 + 3$, $\alpha = -\frac{1}{8}$

$2a+3>0 \to a>-\frac{3}{2}$
$7-2a>0 \to a<\frac{7}{2}$
$a-2<0 \to a<2$

$a \in (2, 3]$, $a=3$ — $y = -\frac{1}{8}x^2 + \frac{1}{8}x$ ... $\frac{+1}{8}$

$c = \frac{1}{8}$

$ax^2 - ax - b = 0$ — α, β ریشه — $40\beta^2 + 20\alpha^2 + 20\beta = 17$

$\alpha+\beta = \frac{a}{a} = 1 \to 20\alpha^2 + 20\beta^2 + 20\beta^2 - 20\beta = 17 \to 20(\alpha^2+\beta^2) + 20\beta(\underbrace{\beta-1}_{-\alpha}) = 17$

$20(1-2\alpha\beta) - 20\alpha\beta = 20 - 40\alpha\beta - 20\alpha\beta = 17 \to 20 - 60\alpha\beta = 17 \to 60\alpha\beta = 3$

$\alpha\beta = \frac{1}{20} = -\frac{b}{a} \to a = -20b$, $-20bx^2 + 20bx - b$ ۲

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{400b^2 - 4\times 20b^2}}{|-20b|} = \frac{\sqrt{320b^2}}{20|b|} = \frac{8\sqrt{5}|b|}{20|b|} = \frac{2\sqrt{5}}{5}$

$(-5, \beta), (1, \beta)$ — $x_S = \frac{1-5}{2} = -2$, $S\left|\begin{matrix}-2\\-\frac{1}{2}\end{matrix}\right. \to y = ax^2 + bx + c$

$y_S = -\frac{1}{2}$ — $-\frac{b}{2a} = -2 \to b = 4a$

$(0, \frac{3}{2})$

$x=-2$, $y=-\frac{1}{2}$ $\to 4a - 2b + \frac{3}{2} = -\frac{1}{2} \to 4a - 2b = -2 \to$ ۲

$y = \frac{1}{2}x^2 + 2x + \frac{3}{2} \to \beta = \frac{1}{2} + 2 + \frac{3}{2} = 4$

$3\alpha^2 + 2\beta^2 = \frac{5}{2}(\alpha^2+\beta^2) + \frac{1}{2}(\alpha^2 - \beta^2) = 12\sqrt{2} + 85$

$\frac{5}{2}(S^2 - 2P) + \frac{1}{2}(S)\left(\frac{\sqrt{\Delta}}{|a|}\right) = 12\sqrt{2} + 85$

$\frac{5}{2}(36 - 2a) + \frac{1}{2}(-6)(\sqrt{36 - 4a}) = 12\sqrt{2} + 85$ ۲

$90 - 5a + 3\sqrt{36 - 4a} = 12\sqrt{2} + 85 \to 90 - 5a = 85 \to a = 1$

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5$ — $36x^2 - (m+14)x + 1 = 0$

$\alpha + \beta = \frac{m+14}{36}$, $\alpha\beta = \frac{1}{36}$

$\frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = 5 \to \frac{\alpha+\beta+2\sqrt{\alpha\beta}}{\alpha\beta} = 25 \to \frac{\frac{m+14}{36} + \frac{2}{6}}{\frac{1}{36}} = \frac{\frac{m+14+12}{36}}{\frac{1}{36}} = 25$

$m + 26 = 25$
$m = -1$

$mx^2 + 2x + 2 = -x^2 + 2x + 2 = 0 \to \boxed{\frac{c}{a} = -2}$ ۲