

$A(2a+3, a-2)$ و $B(7-2a, a-2)$ و $S(b, b-2)$

میانگین طول اضلاع = $\frac{2a+3+7-2a}{2} = 5 \Rightarrow b=5 \Rightarrow b-2=5-2=3 \Rightarrow$

$S(5, 3) \Rightarrow y = k(x-a)^2 + c \Rightarrow 3 - 2a > 0$ و $a - 2 > 0 \Rightarrow a > 2$ و $a < 5$

$a=3 \Rightarrow B=(1, 1)$ و $A=(4, 1) \Rightarrow 1 = k(4-3)^2 + c \Rightarrow 1 = k + c \Rightarrow k = -1 \Rightarrow c = 2$

$y = \frac{-1}{1}(x-3)^2 + 2 \Rightarrow y = -\frac{1}{1}(x-3)^2 + 2 = -\frac{1}{1}(x^2 - 6x + 9) + 2 = -x^2 + 6x - 9 + 2 = -x^2 + 6x - 7$

$\alpha + \beta = \frac{-a}{a} = (-1) \Rightarrow \alpha = -\beta + 1 \Rightarrow 4\alpha^2 + 10\alpha - 20\beta = 17 \Rightarrow 4\beta^2 + \alpha^2 - \beta = \frac{17}{10}$

$4\beta^2 + (1-\beta)^2 - \beta = \frac{17}{10} = 4\beta^2 - 2\beta + 1 + \frac{10}{10} \Rightarrow 4\beta^2 - 2\beta + \frac{11}{10} = 0 \Rightarrow \Delta = 4 - \frac{48}{10} = \frac{16}{10} \Rightarrow$

$\beta = \frac{2 \pm \sqrt{\frac{16}{10}}}{8} = \frac{2 \pm \frac{4}{\sqrt{10}}}{8} = \frac{\sqrt{10} \pm 2}{2\sqrt{10}} = \frac{\Delta \pm 2\sqrt{\Delta}}{10}$

اختلاف اضلاع: $|\alpha - \beta| = \frac{\sqrt{\Delta}}{10} \Rightarrow |(1-\beta) - \beta| = |1-2\beta| \Rightarrow |1-2(\frac{\Delta+2\sqrt{\Delta}}{10})| = |\frac{10-2\Delta-4\sqrt{\Delta}}{10}|$

$y = a(x-h)^2 + k$ و $a = ?$ و $h = \text{طول اضلاع} = \frac{1+(-5)}{2} = -2$

$\Rightarrow y = a(x+2)^2 - \frac{1}{4} \Rightarrow \frac{3}{4} = a(0+2)^2 - \frac{1}{4} \Rightarrow 4a = \frac{3+1}{2} = 2 \Rightarrow a = \frac{1}{2} \Rightarrow$

$\beta = \frac{1}{2}(\frac{-5+2}{2})^2 - \frac{1}{4} = \frac{1}{2}(\frac{-3}{2})^2 - \frac{1}{4} = \frac{9}{8} - \frac{2}{8} = \frac{7}{8}$

$\beta = \frac{1}{2}(\frac{1+2}{2})^2 - \frac{1}{4} = \frac{9}{8} - \frac{2}{8} = \frac{7}{8}$

مادر معادله $4\alpha^2 + 10\alpha - 20\beta = 17$ را در $\alpha = m + 2\sqrt{p}$ و $\beta = m - 2\sqrt{p}$ قرار دهیم

$\alpha = m + 2\sqrt{p}$ و $\beta = m - 2\sqrt{p} \Rightarrow m + 2\sqrt{p} + m - 2\sqrt{p} = -6 \Rightarrow 2m = -6 \Rightarrow m = -3 \Rightarrow$

$\alpha = -3 + 2\sqrt{p}$ و $\beta = -3 - 2\sqrt{p} \Rightarrow 4\alpha^2 + 10\alpha - 20\beta = 17 \Rightarrow 4(-3+2\sqrt{p})^2 + 10(-3+2\sqrt{p}) - 20(-3-2\sqrt{p}) = 17$

$4(9 - 12\sqrt{p} + 4p) - 30 + 20\sqrt{p} + 60 + 40\sqrt{p} = 17 \Rightarrow 36 - 48\sqrt{p} + 16p - 30 + 20\sqrt{p} + 60 + 40\sqrt{p} = 17$

$(\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}})^2 = 2\omega \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 2\omega \Rightarrow \frac{\alpha+\beta}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 2\omega$

$\frac{s}{p} + 2\sqrt{\frac{1}{p}} = 2\omega$ $p = \frac{c}{a} = \frac{1}{4}$ و $s = -\frac{b}{a} = \frac{m+1}{4}$ $\frac{m+1}{4} + 2\sqrt{\frac{1}{4}} = 2\omega \Rightarrow$

$m+1 + 2 \times 2 = 2\omega \Rightarrow m = 2\omega - 4 = -1 \Rightarrow m = -1 \Rightarrow$

$m\alpha^2 + 3\alpha + 2 = 0 \Rightarrow -\alpha^2 + 3\alpha + 2 = 0 \Rightarrow \alpha = 4$ و $\beta = -\frac{2}{4} = -\frac{1}{2}$

حل نهایی: $y = -x^2 + 6x - 7$