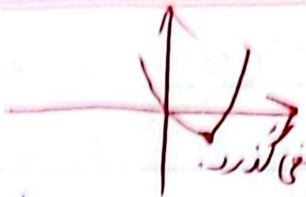
۱)

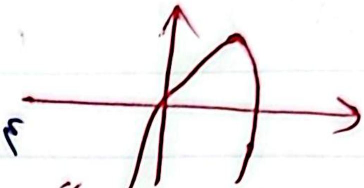


الف) $y = 3x^2 - 2x \Rightarrow a > 0$

$x = \frac{-b}{2a} \Rightarrow \frac{2}{6} = \frac{1}{3} \Rightarrow y = 3 \times \frac{1}{9} - \frac{2}{3} = -\frac{1}{3}$

ب) $y = -x^2 + 4x \Rightarrow a < 0$

$x = \frac{-b}{2a} \Rightarrow \frac{-4}{-2} = 2 \Rightarrow y = -4 + 8 = 4$

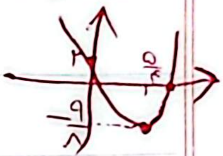


از ناحیه ۲ نمی‌گذرد.

۲_ از ناحیه ۱و۲و۴ نمی‌گذرد.

الف) $y = 2x^2 - 5x + 2 \Rightarrow a > 0$

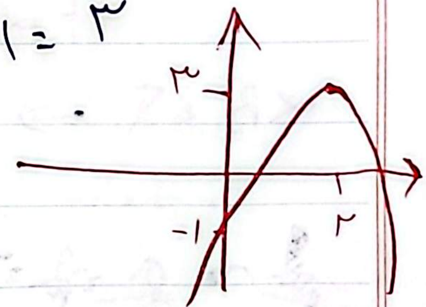
$x = \frac{5}{4} \Rightarrow y = \frac{4ac - b^2}{4a} \Rightarrow \frac{16 - 25}{8} = -\frac{9}{8}$



ب) $y = -x^2 + 4x - 1 \Rightarrow a < 0$

$x = \frac{-b}{2a} \Rightarrow \frac{-4}{-2} = 2 \Rightarrow y = -4 + 8 - 1 = 3$

از ناحیه ۱ و ۳ و ۴ نمی‌گذرد.



۳_ $x^2 - x - 3 = 0 \Rightarrow S = -\frac{b}{a} = 1$ و $P = \frac{c}{a} = -3$

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \sqrt{13}$

الف) $\frac{\alpha + \beta}{\alpha - \beta} = \frac{1}{\sqrt{13}}$

ب) $\alpha^2 + \beta^2 = S^2 - 2P = (\alpha+\beta)^2 - 2(\alpha\beta) = 7$

د) $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = 4\sqrt{13}$

ج) $\alpha^3 + \beta^3 = S^3 - 3PS = (\alpha+\beta)^3 - 3(\alpha\beta)(\alpha+\beta) = 1 + 9 = 10$

۴_ معادله ریشه مضاعف دارد. $y=(x-2)(x^2-ax+a)$

$\Delta<0$ پس $\Rightarrow a^2-4a<0 \Rightarrow a(a-4)<0$

$0<a<4$

همچنین $x=2$ مضاعف است: $y=(x-2)(x-2)^2 \Rightarrow$

$0<a<4 \Leftarrow$ اگر $a=4 \Rightarrow x^2-4x+4=(x-2)^2 \Leftarrow$

$0<a\leq 4$

۵_ $3x^2-12x-a=0 \Rightarrow \alpha+\beta=4$ و $\alpha\beta=-\frac{a}{3}$

$2\alpha^2+\beta^2-4\alpha=7 \Rightarrow \beta=4-\alpha \Rightarrow 2\alpha^2+(4-\alpha)^2-4\alpha=7 \Rightarrow$

$3\alpha^2-12\alpha+9=0$

$\alpha^2-4\alpha+3=0$

$a+b+c=0$

$\alpha=3$ پس $\alpha=1$ یا $3 \Leftarrow$

if $\alpha=3 \Rightarrow \beta=1$

if $\alpha=1 \Rightarrow \beta=3$

$\alpha\beta=3 \Rightarrow 3=-\frac{a}{3} \Rightarrow a=-9$

بیشترین مقدار $=3 \Leftarrow -\frac{a}{3}=\frac{-9}{3}=\boxed{-3}$

۶_ مولفه طبیعی $\Leftarrow \mathbb{N}=\{1,2,3,\dots\}$

$A(2a+3, a-2)$

$B=(7-2a, a-2)$

$\Leftarrow$ و ۲ غ ق ق $\Leftarrow a=3 \Rightarrow A(9,1), B(1,1)$

$S(b, b-2) \Rightarrow b=5 \Rightarrow y_S=3$

$y=a(x-x_S)^2+y_S=a(x-5)^2+3 \xrightarrow{B(1,1)} a(1-5)^2+3=1$

$y=-\frac{1}{8}(x-5)^2+3 \xrightarrow{x=0} -\frac{1}{8}\times 25+3=-\frac{1}{8} \Leftarrow a=-\frac{1}{8}$

فاصله تا محور $=\boxed{+\frac{1}{8}}$

$ax^2 - ax - b = 0 \begin{cases} P = \frac{-b}{a} \\ S = \frac{a}{a} = 1 \end{cases}$ —۷

$30\beta^2 + 30\alpha^2 - 30\beta = 17 \Rightarrow 30\beta^2 + 20\alpha^2 + 10\beta^2 - 10\alpha^2 - 30\beta = 17$

$20(\alpha^2 + \beta^2) + 10(\alpha + \beta)(\beta - \alpha) - 30\beta = 17$

$1 + \frac{2b}{a}$

$20 + \frac{40b}{a} - 10(\alpha + \beta) = 17$

$\frac{40b}{a} = -3 \Rightarrow 40b = -3a$ $\quad b = -\frac{3}{40}a$

$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} \rightarrow \frac{\sqrt{a^2 + 4ab}}{|a|} = \sqrt{1 - \frac{3}{10}} = \sqrt{\frac{7}{10}}$

$x_S = \frac{-a+1}{2} = -2 \Rightarrow f(x) = a(x+2)^2 - \frac{1}{2}$ —۸

$f(x) = \frac{3}{2}$, $x = 0$

$(1, \beta) \in f(x) \Leftarrow \frac{3}{2} = a(0+2)^2 - \frac{1}{2}$

$\Rightarrow \frac{1}{2}(1+2)^2 - \frac{1}{2} = \beta \Rightarrow \beta = 4$

$x^2 + 4x + a \Rightarrow \alpha < \beta < 0$ آنگاه $\begin{cases} P > 0 \Rightarrow P = a \Rightarrow a > 0 \\ S < 0 \Rightarrow S = -4 \end{cases}$ —۹

$2\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85$

$\alpha^2 + \beta^2 = 36 - 2\alpha$

$\alpha^2 + 2(\alpha^2 + \beta^2) = 12\sqrt{2} + 85 \Rightarrow \alpha^2 = 13 + 12\sqrt{2} + 4a$

$72 - 4a$

$\alpha^2 = 18 - a + 4\sqrt{9 - a}$

$= 18 - a + 2\sqrt{36 - 4a}$

$13 + 12\sqrt{2} + 4a \Rightarrow 5 - a = 4a \Rightarrow a = 1$

$\sqrt{36 - 4a} = \sqrt{32} \Rightarrow a = 1$

$$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5 \xrightarrow{\text{به توان ۲}} \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$$

$$\frac{\alpha+\beta}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = \frac{9}{P} + 2\sqrt{\frac{1}{P}}$$

$mx^2+3x+2=0 \rightarrow$ ضرب در -1 $\Rightarrow -x^2+3x+2$

$$\frac{-b}{a} + 2\sqrt{\frac{a}{c}} = \frac{m+16}{1} + 2\sqrt{\frac{m+24}{1}} = m+24 \Rightarrow m+24 = 25$$

$$m = -1$$