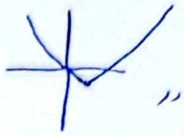
پویا تحصیلی   تکلیف ۲۵   کلاس دهم پسر

۱-

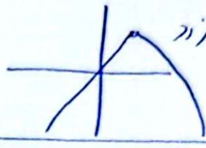
الف) $y = 3x^2 - 2x \rightarrow a > 0$ $x_s = -\frac{-2}{6} = \frac{1}{3}$ $y_s = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$



از ناحیه ۳ نمی‌گذرد

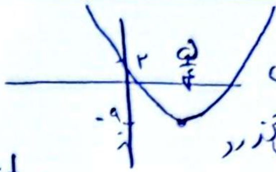
ب) $y = -x^2 + 4x \rightarrow a < 0$ $x_s = -\frac{4}{-2} = 2$ $y_s = -4 + 8 = 4$

از ناحیه ۲ نمی‌گذرد



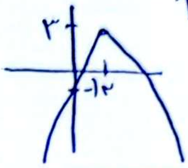
۲-

الف) $y = 2x^2 - 5x + 2 \rightarrow a > 0$ $x_s = \frac{5}{4}$ $y_s = -\frac{9}{8}$



از ناحیه‌های ۱، ۲ و ۴ می‌گذرد

ب) $y = -x^2 + 4x - 1$ $a < 0$ $x_s = 2$ $y_s = 3$



از ناحیه‌های ۱ و ۳ و ۴ می‌گذرد

۳-

$x^2 - x - 3 = 0$ α, β ریشه $S = 1$ $P = -3$ $\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \sqrt{13}$

الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

ب) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 1 + 6 = 7$

ج) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta) = 10$

د) $\alpha^3 - \beta^3 = (\alpha-\beta)(\alpha^2 + \alpha\beta + \beta^2) = 4\sqrt{13}$

۴-

$y = (x-2)(x^2 - ax + a)$ ← ریشه مضاعف

$x - 2 = 0 \rightarrow x = 2$

$\Delta < 0 \rightarrow a^2 - 4a < 0 \rightarrow a(a-4) < 0 \rightarrow 0 < a < 4$

if $a = 4 \rightarrow x^2 - 4x + 4 = (x-2)^2$ ✓

چون ریشه $x - 2$ هم ۲ است پس باز هم مضاعف

$\rightarrow 0 < a \leq 4$

۵-

$3x^2 - 12x - a = 0 \rightarrow \alpha + \beta = 4$ $\alpha\beta = -\frac{a}{3}$

$2\alpha^2 + \beta^2 - 4\alpha = 7 \xrightarrow{\beta = 4 - \alpha} 2\alpha^2 + (4 - \alpha)^2 - 4\alpha = 7 \rightarrow 3\alpha^2 - 12\alpha + 9 = 0$

$\rightarrow \alpha^2 - 4\alpha + 3 = 0 \rightarrow (\alpha - 1)(\alpha - 3) = 0$ → $\alpha = 1, \beta = 3$ یا $\beta = 1, \alpha = 3$

$\alpha\beta = 3 = -\frac{a}{3} \Rightarrow a = -9$

$\frac{a}{\text{ریشه بزرگتر}} = \frac{-9}{3} = -3$

۶-

$A(2a, 3, a-2)$

$B(7-2a, a-2) \xrightarrow[a=3]{\text{عرض برابر}} B(1,1)$, $x_s = \frac{x_A + x_B}{2} = \frac{2a+3+7-2a}{2} = 5 \Rightarrow b = 5$

$S(b, b-2) \xrightarrow{b=5} S(5,3)$

$y = a(x-x_s)^2 + y_s = a(x-5)^2 + 3 \xrightarrow{B(1,1)} a(1-5)^2 + 3 = 1 \Rightarrow a = -\frac{1}{8}$

$y = -\frac{1}{8}(x-5)^2 + 3 \xrightarrow{x=0} y = -\frac{25}{8} + 3 = -\frac{1}{8} \rightarrow$ فاصله تا مبدا عرض‌ها $= \frac{1}{8}$

۷-

$ax^2 - ax - b = 0 \rightarrow S = -\frac{-a}{a} = 1 \quad P = -\frac{b}{a}$

~~$4\alpha\beta^2 + 2\alpha x^2 - 2\alpha\beta = 17$~~ $\rightarrow 30\beta^2 + 30\alpha^2 + 10\beta^2 - 10\alpha^2 - 20\beta = 17 \rightarrow 30(\alpha^2+\beta^2) + 10(\beta-\alpha)^2 - 20\alpha\beta = 17$

$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2+4ab}}{|a|} = \sqrt{1-\frac{1}{5}} = \frac{2}{\sqrt{5}}$ $\Rightarrow 30 + \frac{60b}{a} - 10(\alpha\beta) = 17$

$\rightarrow 60b = -3a \Rightarrow b = -\frac{1}{20}a$

۸-

$y = ax^2 + bx + c \rightarrow c = \frac{3}{4}$

$y_s = \frac{1}{4}$ $\quad (-5, \beta), (1, \beta) \xrightarrow{\text{عرض مشترک دارند}} \frac{-5+1}{2} = x_s = -2$

$y = a(x+2)^2 - \frac{1}{4} \xrightarrow{(0, \frac{3}{4})} a(0+2)^2 - \frac{1}{4} = \frac{3}{4} \rightarrow a = \frac{1}{4}$

$(1, \beta) \in f(x) \rightarrow \frac{1}{4}(1+2)^2 - \frac{1}{4} = \beta \Rightarrow \beta = 2$

۹-

$x^2 + 6x + a = 0 \quad \alpha < \beta < 0 \rightarrow S < 0 \quad P > 0$

α و β منفی، $S = -6$

$P = a \Rightarrow a > 0 \quad \Delta = 36 - 4a \quad x = \frac{-6 \pm 2\sqrt{9-a}}{2} \quad \alpha = -3 + \sqrt{9-a} \quad \beta = -3 - \sqrt{9-a}$

$3\alpha^2 + 2\beta^2 = 85 + 12\sqrt{2}$

$\alpha^2 + \underbrace{2(\alpha^2+\beta^2)}_{72-4a} = 85 + 12\sqrt{2} \rightarrow \alpha^2 = 13 + 12\sqrt{2} + 4a \rightarrow \alpha^2 = 18 - a + 6\sqrt{9-a} = 18 - a + 3\sqrt{36-4a}$

$18 - a + 3\sqrt{36-4a} = 13 + 12\sqrt{2} + 4a \rightarrow 5 - a = 4a \Rightarrow a = 1$ ، $3\sqrt{36-4a} = 12\sqrt{2} \Rightarrow a = 1$

۱۰-

$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5 \rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25 \rightarrow \frac{\alpha+\beta}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$mx^2 + 3x + 2 = 0 \quad 36x^2 - (m+14)x + 1 = 0$ $\quad = \frac{S}{P} + 2\sqrt{\frac{1}{P}} = -\frac{b}{c} + 2\sqrt{\frac{a}{c}}$

$= m + 14 + 2\sqrt{36} \Rightarrow m = -1$

$\rightarrow -x^2 + 3x + 2 = 0 \rightarrow \frac{c}{a} = -2$