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نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۴ کلاس دهم B .....

۱

$x^2 - ax + b$ $\quad \begin{array}{c|cc} & 1 & 3 \\ \hline +4 & - & + \end{array}$ $\Rightarrow x^2 - 5x + 6 = x^2 - 4x + c$

$\Rightarrow a = +4 \quad b = 3 \quad \Rightarrow a + b = +7$ ✓ (۴)

۲

$y = ((k-2)x + m - 1)(x - 4n)^2 \Rightarrow x - 4n = 0 \Rightarrow x = 4n = -1 \Rightarrow n = -\frac{1}{4}$

$\Rightarrow y = ((k-2)x + m - 1)(x + 1)^2 \Rightarrow 4k - 9 + m = 0 \quad , \quad (k-2) < 0$

$\Rightarrow k < 2$ طبیعی $\Rightarrow k = 1 \Rightarrow m = 9 \Rightarrow \frac{m}{4} + k = -|4| =$ ✓ (۴)

۳

$y = -\frac{1}{2}x^2 + 2x + 4 \Rightarrow -\frac{1}{2}x^2 + 2x + 4 > \frac{7}{2}$

$\Rightarrow -\frac{1}{2}x^2 + 2x + \frac{9}{2} > 0 \Rightarrow \begin{array}{c|cc} & -1 & 9 \\ \hline - & + & - \end{array} \Rightarrow (-1, 9)$ (۲)

$\Rightarrow a = -1 \quad b = 9 \Rightarrow \max \quad b - ac = \frac{49}{2}$ ✓

۴

$f(x) = x^3 - 3x^2 - x + 3 \Rightarrow a + b + c + d = 0 \Rightarrow x = 1$

$\Rightarrow x^3 - 4x^2 - x + 4 \div x - 1 = x^2 - 3x - 4 \Rightarrow (x-1)(x^2 - 3x - 4) = f(x)$

$\Rightarrow \begin{array}{c|ccc} & -1 & 1 & 4 \\ \hline - & + & - & + \end{array} \Rightarrow (-\infty, -1)$ و $(1, 4)$ (۲)

$\Rightarrow \frac{1+4}{2} = 2$ ✓

$\Rightarrow f(2) = 8 - 12 - 2 + 3 = -3$

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$(a-1)x^2 + (a-1)x + 1 < 0 \Rightarrow \Delta < 0 \quad , \quad (a-1) < 0$

$\Rightarrow \Delta < 0 : (a-1)^2 - 4(a-1) < 0 \Rightarrow a^2 - 5a + 5 < 0 \Rightarrow \begin{array}{c|cc} & 1 & 5 \\ \hline + & - & + \end{array} \quad (1, 5)$ (۲)

$(a-1) < 0 \Rightarrow a < 1$

$\Rightarrow (-\infty, 1) \cap (1, 5) = \emptyset$ ✓

۶

$\frac{m(m^3+m)}{m-2} > 0 \Rightarrow \frac{m^2(m^2+1)}{m-2} > 0$

	$-\infty$		2		$+\infty$
m^2	+	0	+	+	
m^2+1	+	+	+	+	
$m-2$	−	−	0	+	
$\frac{m^2(m^2+1)}{m-2}$	−	0	+	+	

$\Rightarrow \{x \mid x \in (2, +\infty)\}$

$m(m^3+m) = m(m(m^2+1)) = m^2(m^2+1)$ → همواره مثبت

$\frac{m(m^3+m)}{m-2} > 0$ → صورت همواره مثبت → $m-2 > 0$ → $m > 2$

(۱)

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$\frac{(x^2-x-4)(x-1)^4}{(x^2+x+1)(4-x)^3} \leq 0 \Rightarrow \frac{(x-4)(x+1)(x-1)^4}{(x^2+x+1)(x-4)^3} \leq 0$

همواره مثبت

	$-\infty$		-1		1		4		$+\infty$
$x-4$	−	−	−	−	−	−	0	+	
$x+1$	−	−	0	+	+	+	+	+	
$(x-1)^4$	+	+	+	+	0	+	+	+	
x^2+x+1	+	+	+	+	+	+	+	+	
$(x-4)^3$	−	−	−	−	−	−	0	+	
$\frac{(x-4)(x+1)(x-1)^4}{(x^2+x+1)(x-4)^3}$	+	+	0	−	0	+	‖	−	

$\Rightarrow \{x \mid x \in [-1, 4) \cup [4, +\infty)\}$

(۲)

۸

$\frac{2x^2-2x}{x^2+4} < 2 \Rightarrow \frac{2x^2-2x}{x^2+4} - 2 < 0 \Rightarrow \frac{2x^2-2x-2x^2-8}{x^2+4} < 0$

$\Rightarrow \frac{-2x-8}{x^2+4} < 0 \Rightarrow \frac{-2(x+4)}{x^2+4} < 0$

	$-\infty$		-4		$+\infty$
-2	−	−	−	−	
$x+4$	−	−	0	+	
x^2+4	+	+	+	+	
$\frac{-2(x+4)}{x^2+4}$	+	+	0	−	

$\Rightarrow (-4, +\infty)$

(۲)

$b-a = 4-(-4) = 8$

۹

$-1 < \frac{x^3-4x}{x+1} < 0 \Rightarrow \frac{x^3-4x}{x+1} < 0 \Rightarrow \frac{x(x^2-4)}{x+1} < 0$

	$-\infty$		-2		-1		0		2		$+\infty$
x	−	−	−	−	−	−	0	+	+	+	
x^2-4	+	+	0	−	−	−	−	−	0	+	
$x+1$	−	−	−	−	0	+	+	+	+	+	
$\frac{x(x^2-4)}{x+1}$	−	0	+	0	−	‖	+	0	−	0	

$(-\infty, -2) \cup (-1, 0) \cup (2, +\infty)$

$\frac{x^3-4x}{x+1} > -1 \Rightarrow \frac{x^3-4x+x+1}{x+1} > 0 \Rightarrow \frac{x^3-3x+1}{x+1} > 0$

	$-\infty$		-1		$\frac{-1}{2}$		$\frac{1}{2}$		$+\infty$
$x+1$	−	−	0	+	+	+	+	+	
x^3-3x+1	−	−	−	−	0	+	0	−	
$\frac{x^3-3x+1}{x+1}$	+	‖	−	‖	0	+	0	−	

$(-1, \frac{-1}{2}) \cup (1, +\infty)$

$\Rightarrow (-\infty, -1) \cup (-1, \frac{-1}{2}) \cup (1, +\infty)$

$= (-\infty, \frac{-1}{2}) \cup (1, +\infty)$

(۱)

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$\frac{x^2-10}{x} \leq 3 \Rightarrow \frac{x^2-10}{x} - 3 \leq 0 \Rightarrow \frac{x^2-3x-10}{x} \leq 0$

	$-\infty$		-2		0		5		$+\infty$
$x^2-3x-10$	+	+	0	−	−	−	0	+	
x	−	−	−	−	0	+	+	+	
$\frac{x^2-3x-10}{x}$	−	‖	+	0	−	‖	+	0	

$\Rightarrow \{x \mid x \in (-\infty, -2] \cup (0, 5]\}$

(۲)

$-1 < \frac{x^3-4x}{x+1} \rightarrow \frac{x^3-4x}{x+1} + 1 > 0 \rightarrow \frac{x^3-4x+x+1}{x+1} > 0 \rightarrow \frac{x^3-3x+1}{x+1} > 0$

صورت همواره مثبت → $x+1 > 0 \rightarrow x > -1$ (I)

(I) ∧ (II) → $0 < x < \frac{1}{3}$