۱-

$y = ax^2 - 9x + b$

$\begin{array}{ccccc} & 1 & & 3 & \\ + & 0 & - & 0 & + \end{array} \to y = (x-1)(x-3) = x^2 - 4x + 3 \quad a = 4 \quad b = 3$

$a + b = 7$

۲

۲-

$y = ((k-2)x + m - 1)(x - 3n)^2$

$\begin{array}{ccccc} & -1 & & 4 & \\ + & 0 & + & 0 & - \end{array} \to \text{if } x = -1 \to x - 3n = 0 \Rightarrow n = -\frac{1}{3}$

$\to k - 2 < 0 \to k < 2, \; k \in \mathbb{N} \to k = 1$

$\text{if } x = 4 \to 4(k-2) + m - 1 = 0 \to 4k + m - 9 = 0$

$\Rightarrow m - 5 = 0 \Rightarrow m = 5$

$\frac{m}{n} + k = \frac{5}{-\frac{1}{3}} + 1 = -14$

۲

۳-

$y = -\frac{1}{2}x^2 + 2x + 6 \to -\frac{1}{2}x^2 + 2x + 6 > \frac{7}{2} \to -\frac{1}{2}x^2 + 2x + \frac{5}{2} > 0 \xrightarrow{\times(-2)} x^2 - 4x - 5 < 0$

$\to (x-5)(x+1) < 0 \begin{cases} \to a = -1 \\ \to b = 5 \end{cases} \quad b - a = 5 - (-1) = 6$

۲

۴-

$f(x) = x^3 - 3x^2 - x + 3 \quad x > 0$

$\to f(x) = x^2(x-3) - 1(x-3) \to f(x) = (x^2-1)(x-3) \xrightarrow[x > 0]{f(x) < 0} (a,b) \to a = 1, \; b = 3$

نقطه میانی بازه (a,b) $\to \frac{a+b}{2} \to \frac{4}{2} = 2 \to f(2) = (4-1)(2-3) = -3$

۲

۵-

$(a-1)x^2 + (a-1)x + 1 < 0 \to a - 1 < 0 \to a \in (-\infty, 1)$

$\Delta < 0 \to a^2 - 2a + 1 - 4a + 4 < 0 \to a^2 - 6a + 5 < 0$

$\to (a-1)(a-5) < 0 \quad \begin{array}{ccccc} & 1 & & 5 & \\ + & 0 & - & 0 & + \end{array} \to a \in (1, 5)$

$(-\infty, 1) \cap (1, 5) = \emptyset$

۲

۶-

$\frac{m(m^3 + m)}{m - 2} > 0 \to \frac{m^2 \overbrace{(m^2+1)}^{\text{همواره مثبت}}}{m - 2} > 0 \quad \begin{array}{ccccc} & 0^* & & 2 & \\ - & & - & \text{ت} & + \end{array} \quad m \in (2, +\infty)$

۲

۷-

$\frac{(x^2 - x - 6)(x-1)^2}{(x^2+x+1)(2-x)^3} \leq 0 \to \frac{(x-3)(x+2)(x-1)^2}{\underbrace{(x^2+x+1)}_{\text{ریشه ندارد}}(2-x)^3} \quad \begin{array}{ccccccccc} & -2 & & 1^* & & 2 & & 3 & \\ + & 0 & - & 0 & - & \text{ت} & + & 0 & - \end{array}$

$[-2, 2) \cup [3, +\infty)$

۲

۸-

$f(x) = \frac{3x^2-2x}{x^2+4}$

$f(x) < 2 \longrightarrow \frac{3x^2-2x}{x^2+4} < 2 \longrightarrow \frac{3x^2-2x-2x^2-8}{x^2+4} < 0$

$\longrightarrow \frac{x^2-2x-8}{x^2+4} < 0 \longrightarrow \frac{(x-4)(x+2)}{x^2+4} < 0$

x^2+4 همواره مثبت

-2		4	
$+$	$-$		$+$

$(a,b) = (-2,4)$

$b-a = 4-(-2) = 6$

۲

۹-

$-1 < \frac{3x^2-4x}{x+1} < 0$

$\frac{3x^2-4x}{x+1} > -1 \longrightarrow \frac{3x^2-4x+x+1}{x+1} > 0 \longrightarrow \frac{3x^2-3x+1}{x+1} > 0$

$3x^2-3x+1$ همواره مثبت

① $(-1,+\infty)$

$\frac{3x^2-4x}{x+1} < 0 \longrightarrow \frac{x(3x-4)}{x+1} < 0$

-1		0		$\frac{4}{3}$	
$-$		$+$		$-$	$+$

② $(-\infty,-1) \cup (0,\frac{4}{3})$

① $\cap$ ② $= (0,\frac{4}{3})$

۲

۱۰-

$\frac{x^2-10}{x} \le 3 \longrightarrow \frac{x^2-3x-10}{x} \le 0 \longrightarrow \frac{(x-5)(x+2)}{x} \le 0$

-2		0		5	
$-$		$+$		$-$	$+$

$x \in (-\infty,-2] \cup (0,5]$

۲