

1) $m \in \mathbb{Z}$, $m^2 - a^2 + b$, $a, b \in \mathbb{Z}$
 $\Delta < 0$
 $\hookrightarrow m > 3, m < 1 \rightarrow$ نیست $\begin{cases} m=1 \\ m=3 \end{cases}$ $\begin{cases} 2-3a+b=0 \\ 1-a+b=0 \end{cases} \Rightarrow \begin{cases} 3a-b=9 \\ -a+b=-1 \end{cases}$
 $\underline{2a=8}$
 $\Rightarrow a=4, b=3$ $4+3=7$ ✓

2) $y = (k-2)(n+m-1)(n-3n)^2$ $\frac{-4k}{+b} + \frac{4}{b-} = \frac{m}{n} + k = ?$
 $3n = -1 \Rightarrow 2+n = -\frac{1}{3} \Rightarrow \frac{1-m}{k-2} = 4 \Rightarrow 4k-8 = 1-m \Rightarrow m = 9-4k$
 $\Rightarrow y = (k-2)(n-4)(n+1)^2 \Rightarrow m=5, k=1, n=-\frac{1}{3} \Rightarrow (5, (-\frac{1}{3}), 1)$
 (-14) ✓

3) $y = -\frac{1}{2}n^2 + 2n + 6$ $(a, b) \rightarrow \frac{7}{2}$
 $-\frac{1}{2}n^2 + 2n + 6 > \frac{7}{2} \Rightarrow -n^2 + 4n + 7 > 7 \Rightarrow -n^2 + 4n + 5 > 0$
 $\frac{-4 \pm \sqrt{16+20}}{-2} = \begin{cases} \frac{4-6}{-2} = 5 \rightarrow b \\ \frac{4+6}{-2} = -1 \rightarrow a \end{cases}$ $\max(b-a) \rightarrow 5 - (-1) = 6$ ✓

4) $f(n) = n^3 - 3n^2 + n + 3 = (n+1)(n-1)(n-3)$
 $\Rightarrow (1, 3)$ $\Rightarrow f(2) = 8 - 12 - 2 + 3 = -3$ ✓

5) $(a-1)n^2 + (a-1)n + 1 \rightarrow$ نیست $a-1 < 0 \Rightarrow a < 1$
 $\Delta < 0 \Rightarrow a^2 - 2a + 1 - 4a + 4 < 0 \Rightarrow a^2 - 6a + 5 < 0 \Rightarrow (a-1)(a-5) < 0$
 $\frac{1}{-} \frac{5}{+} \Rightarrow a \in (1, 5)$ ✓

$$\frac{m(m^3+m)}{m-2} > 0 \Rightarrow \frac{m(m^3+m)}{m-2} > 0 \Rightarrow \frac{m^2(m^2+1)}{m-2} > 0 \Rightarrow \frac{+}{-} \frac{+}{-} \frac{+}{+}$$

$$\Rightarrow m \in (2, +\infty)$$

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$$\frac{(n^2-2n-6)(n-1)^2}{(n^2+n+1)(2-n)^3} \leq 0 \Rightarrow \frac{(n-3)(n+2)(n-1)^2}{(n^2+n+1)(2-n)^3}$$

$\Delta < 0$ (no real roots)

$$\Rightarrow \frac{+}{+} \frac{+}{-} \frac{+}{+} \frac{+}{-} = (-\infty, -2) \cup (3, +\infty)$$

$$\frac{-}{+} \frac{+}{-} \frac{-}{-} \frac{+}{-} = [-2, 3] \cup [3, +\infty)$$

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$$f(x) = \frac{3x^2-2x}{x^2+4} \quad (a, b) \quad y=2$$



$$f(x) = \frac{3x^2-2x}{x^2+4} \quad (2)$$

$$\frac{3x^2-2x}{x^2+4} = 2 \Rightarrow 3x^2-2x = 2x^2+8 \Rightarrow x^2-2x-8 < 0 \Rightarrow \frac{2 \pm \sqrt{4+32}}{2} \quad \frac{8}{2} = 4 \rightarrow b$$

$$\frac{-4}{2} = -2 \rightarrow a$$

$$b-a = 4 - (-2) = 6$$

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$$-1 < \frac{3n^2-4n}{n+1} < 0 \quad \frac{3n^2-4n}{n+1} + \frac{n+1}{n+1} > 0 \Rightarrow \frac{3n^2-3n+1}{n+1} > 0$$

$$\Rightarrow \frac{-1}{-} \frac{+}{+} \Rightarrow (-1, +\infty)$$

$$\frac{3n^2-4n}{n+1} < 0 \Rightarrow \frac{n(3n-4)}{n+1} < 0 \Rightarrow \frac{-}{-} \frac{+}{+} \frac{+}{+} \Rightarrow (-\infty, -1) \cup (0, \frac{4}{3})$$

$$\text{Intersection} \Rightarrow (0, \frac{4}{3})$$

$$\frac{n^2-10}{n} < 3 \Rightarrow \frac{n^2-10}{n} - 3 < 0 \Rightarrow \frac{n^2-10-3n}{n} < 0 \Rightarrow \frac{(n+2)(n-5)}{n} < 0$$

$$\Rightarrow \frac{-}{-} \frac{+}{+} \frac{-}{+} \frac{+}{+} \Rightarrow (-\infty, -2] \cup (0, 5]$$

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