۲۰ امتحان نهایی

$\begin{cases} 3x - y = 9 \\ x + 2y = -4 \end{cases} \rightarrow \begin{cases} 6x - 2y = 18 \\ x + 2y = -4 \end{cases}$

$7x = 14 \rightarrow x = 2$

$6 - 2y = 9 \rightarrow -2y = 3 \rightarrow y = -\frac{3}{2}$

$\frac{x}{y} = \frac{2}{-\frac{3}{2}}$ ✓

$\begin{cases} \frac{1}{x} - \frac{1}{y} = -1 \\ \frac{2}{x} - \frac{3}{y} = -2 \end{cases} \rightarrow \begin{cases} y - x = -xy \\ 2y - 3x = -2xy \end{cases}$

$2x - 3x = -2xy + xy \rightarrow -x = -xy \rightarrow y = 1$

$y - x = -xy \rightarrow 1 - x = -x \rightarrow 1 = 0$ ✗

$\frac{x}{y} = \frac{1}{2}$ ✓ (۲)

$y = 1 \rightarrow x = 2y \rightarrow x = 2$

$x + 2y = 4 \rightarrow 2x + 4y = 8$

$5x = 10 \rightarrow x = 2$ ✓

$f(a) + 2f(2) = 3f(1) \rightarrow a + 2b = -4$; $a + 1 = -3 \rightarrow a = -4$ (۲)

$2(-4) + 2b = -4 \rightarrow b = 0$ ✓

$m^3 - 3m = -2 \rightarrow m^3 - 3m + 2 = 0 \rightarrow (m-1)(m-2) = 0$

$m = 2 \rightarrow$ تابع ثابت $\alpha(3,4)$

$m = 1 \rightarrow$ تابع ثابت $\alpha(3,2)$ (۲)

تابع ثابت $(3,2)$ به ازای هیچ مقدار m

الف) تابع ثابت

ب) تابع همانی

ج) تابع ثابت

د) تابع همانی (۲)

$y = -\sqrt{x+1}$ تابع همانی

$x = \frac{y}{\sqrt{1-y^2}} \cdot \frac{\sqrt{1-y^2}}{y} = \frac{y}{y} \rightarrow \frac{y^2}{1-y^2} = \frac{y^2}{1-y^2}$

$y_1^2 - y_1^2 y_2^2 = y_2^2 - y_1^2 y_2^2 \xrightarrow{y_1 y_2 \neq 0}$ همانی $y_1 = y_2 \rightarrow$ همانی ✓ (۲)

$|y| = x \xrightarrow{x=1} y = \{-1, 1\}$ ✓

$y^3 + y^2 + y + x^2 + x = 0$

$y^3 + y^2 + y = -x^2 - x$ (+) $y = 0$

تابع همانی (۲)

$f(x) = \frac{x^2 + 3x + 2}{x^2 + 3x + 5} \rightarrow \frac{(x+2)(x+1)}{(x+2)^2 + 1} = \frac{x+1}{x+3} = \frac{-2+1}{-2+3} = \frac{-1}{1} = -1$ (۲)

$f(x) = x^3 + ax + b$; $y = 3x - a$

$x = -1 \rightarrow -3 = -1 - a + b \rightarrow -2 = b - a$

$x = 1 \rightarrow -1 = 1 + a + b \rightarrow -2 = a + b$

$-4 = 2b \rightarrow b = -2$; $a = 0$

$x^3 - x = 0 \rightarrow x(x^2 - 1) = 0 \rightarrow x(x+1)(x-1) = 0$

$x = 0, -1, 1$

$x^3 + x^2 - x - 1 = 0 \rightarrow (x^2 - 1)(x+1) = 0$

$x = -1$ (دوگانه) ; $x = 1$ ✓ (۲)

$f(x) = \frac{3x^2 - 5x + 1}{2x + 3} = x \rightarrow 3x^2 - 5x + 1 = 2x^2 + 3x$

$x^2 - 8x + 1 = 0$

$\Delta = 64 - 4 = 60$

$x = \frac{8 \pm \sqrt{60}}{2} = 4 \pm \sqrt{15}$

$x_1 + x_2 = 8$ ✓ (۲)

$a + b = 2a \rightarrow a = b$

$a - 2b + 1 = 2a \rightarrow -a + 1 = 2a \rightarrow 3a = 1 \rightarrow a = \frac{1}{3}$ ✓ (۲)

$f(x) = \frac{3x^2 - 5x + 1}{2x + 3} = x \rightarrow 3x^2 - 5x + 1 = 2x^2 + 3x$

$x^2 - 8x + 1 = 0$

$a = 1$; $b = -8$; $c = 1$

$a + b + c = 1 - 8 + 1 = -6 \neq 0$ ✓ (۲)