

الف)  $(9, x+2y), (x-y, -4)$   $\left. \begin{matrix} 2x-y=9 \\ x+2y=-4 \end{matrix} \right\} \Rightarrow x=2, y=-3$  ۲

$\Rightarrow \frac{x}{y} = -\frac{2}{3}$

ب)  $(-1, -4), (\frac{1}{x} - \frac{1}{y}, \frac{a}{x} - \frac{y}{y})$   $\frac{1}{x} - \frac{1}{y} = -1$   $\frac{a}{x} - \frac{y}{y} = -4$

$\frac{-a}{x} + \frac{y}{y} = 4$   $\frac{a}{x} - \frac{a}{y} = -4 \Rightarrow \frac{y}{y} = -2 \Rightarrow y = -1, x = -\frac{11}{2}$   $\Rightarrow \frac{x}{y} = \frac{1}{2}$

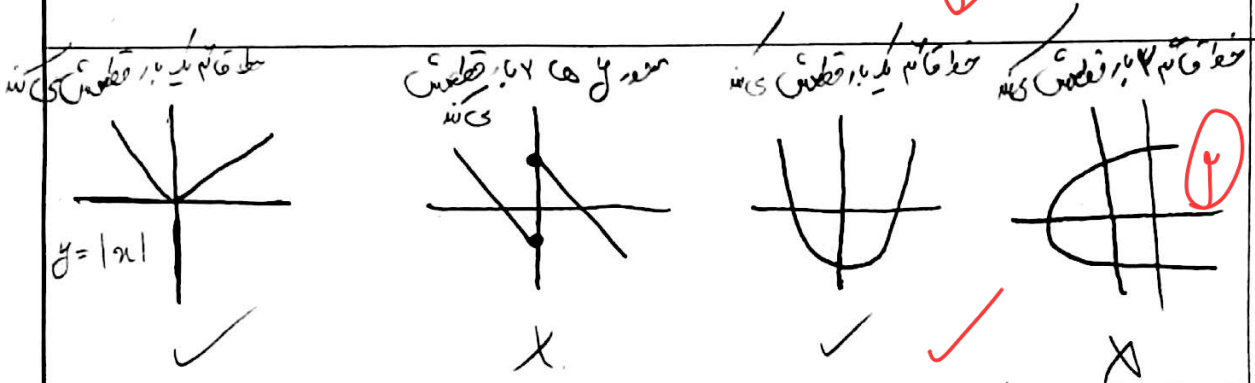
$f = \{(a, 2a), (1, a+1), (1, -2), (2, b)\}$

$f(a) + 2f(1) = 3f(1) \Rightarrow 2a + 2 \times b = 3a + 4 \Rightarrow 2b = a + 4$

$a+1 = -2 \Rightarrow a = -3 \Rightarrow 2b = 0 \Rightarrow b = 0$  ۲

$f = \{(-1, m^2 - 4m), (2, a), (-1, -2), (m+1, 4), (2, 4), (m^2+2, 4m+1)\}$

$m^2 - 4m = -2 \Rightarrow m^2 - 4m + 2 = 0 \Rightarrow m = 1 \text{ و } 3$  ۲



الف)  $y = -\sqrt{x+1}$  ۱۵

طیف اول تابع است

ب)  $x = \frac{y}{\sqrt{1-y}}$   $x=1: \sqrt{1-y} = y$

$\Rightarrow 1-y = y^2 \Rightarrow y^2 + y - 1 = 0$

$\Rightarrow y = \frac{-1 \pm \sqrt{5}}{2}$  X

$x = \frac{y}{\sqrt{1-y^2}} \rightarrow \text{تغییر متغیر: } \begin{cases} x = \frac{y_1}{\sqrt{1-y_1^2}} \\ x = \frac{y_2}{\sqrt{1-y_2^2}} \end{cases} \rightarrow \frac{y_1}{\sqrt{1-y_1^2}} = \frac{y_2}{\sqrt{1-y_2^2}} \rightarrow |y_1| = |y_2|$

$y_1 = y_2 \rightarrow \text{تغییر متغیر}$

الف)  $|y| = x$  حيث  $a = a \Rightarrow |y| = a \Rightarrow y = \pm a x$

ب)  $y'' + \alpha y' + \beta y + \gamma x + \delta = 0$   $x_1 = x_2$  ✓ (P)  
 $\Rightarrow y_1(y_1' + \alpha y_1 + \beta) = y_2(y_2' + \alpha y_2 + \beta) \Rightarrow y_1 = y_2$  ✓ ,  $y_1' + \alpha y_1 + \beta = y_2' + \alpha y_2 + \beta$   
 $\Rightarrow y_1(y_1 + \beta) = y_2(y_2 + \beta) \Rightarrow y_1 = y_2$  ✓ ,  $y_1 + \beta = y_2 + \beta \Rightarrow y_1 = y_2$  ✓ (P)

$f(x) = \frac{x^2 + \epsilon x + \alpha}{x^2 + \epsilon x + \nu} = \frac{(x+\gamma)^2 + 1}{(x+\gamma)^2 + \nu} \Rightarrow \frac{(\sqrt{x})^2 + 1}{(\sqrt{x})^2 + \epsilon}$   
 $= \frac{\nu + 1}{\nu + \epsilon} = \frac{\epsilon}{\nu} = \frac{\nu}{\epsilon}$  ✓ (P)

$x^2 - \alpha x - 1 = (x+1)(x^2 - \alpha x - 1) \dots \rightarrow x^2 - \alpha x - 1 = 0 \rightarrow \Delta = \frac{-b}{a} = 1$   
 $y - \epsilon x + \alpha = 0 \quad (-1, -\epsilon) : \nu + \alpha - \epsilon = 0 \Rightarrow \alpha = 1$  (P)  
 $y = x^2 + \alpha x + \beta \quad (-1, -\epsilon) : -\epsilon = -1 + \beta \Rightarrow \beta = -\nu$   
 $y = x^2 + x - \nu \Rightarrow y = \nu x - 1 \Rightarrow x^2 + x - \nu = \nu x - 1 \Rightarrow x^2 - \nu x + 1 = 0$   
 $\Rightarrow (x-1)(x^2 + x - 1) \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow \Delta = -1$

$f = \{ (1, a+b), (1, \nu a^2), (-1, a - \nu b + 1) \}$   
 $\nu a = a - \nu b + 1 \Rightarrow a + \nu b = 1$   
 $a + b = \nu a \Rightarrow -a + b = 0$  }  $\Rightarrow a = \frac{1}{\nu} \quad b = \frac{1}{\nu}$  (P)

$f(0) = 0 \Rightarrow \frac{\epsilon + 1}{\epsilon} = 0 \Rightarrow \epsilon = -1 \Rightarrow \frac{\epsilon x^2 - a x}{b x + \epsilon} = f(x)$   
 $f(1) = 1 \Rightarrow \frac{\epsilon - a}{b + \epsilon} = 1 \Rightarrow \epsilon - a = b + \epsilon \Rightarrow a + b = 1$  (P)  
 $f(-1) = -1 \Rightarrow \frac{\epsilon + a}{-b + \epsilon} = -1 \Rightarrow \epsilon + a = b - \epsilon \Rightarrow \frac{b - a = \nu}{a = -\nu \quad b = \epsilon}$   
 $\Rightarrow a + b + c = 0$