

$$x^2 + 7x = a(x - 1) \quad \text{if } x=1 \Rightarrow a + 7a = a - 1 \Rightarrow 7a = -1 \Rightarrow a = -\frac{1}{7}$$

(2)

$$g(x) = 7x + b \xrightarrow{x=1} 7 = 1 + b \Rightarrow b = -6$$

$$\Rightarrow f(x) = \frac{x^2 + a}{7x - b} \Rightarrow 1 = \frac{1 + a}{7 - (-6)} \Rightarrow a = 11$$

(2)

$$f(1) = \frac{1 + 11}{7 - (-6)} = \frac{12}{13} = \frac{12}{13}$$

$$\begin{aligned} 7 - a + b &= 0 & -a + b &= -7 & a - b &= 7 \\ 7 + 1 + a + b &= 0 & 8 + a + b &= -1 & a + b &= -9 \end{aligned} \Rightarrow a = -9, b = -1$$

(2)

$$\Rightarrow f(1) = \frac{1 + (-9)}{7 + (-1) - 1} = \frac{-8}{5} = -\frac{8}{5}$$

$$-1 + a(-1) + b = 0 \Rightarrow -1 - a + b = 0 \Rightarrow a - b = -1$$

$$\begin{aligned} \Delta = 0 &\Rightarrow a^2 + 14b = 0 & \Rightarrow 14a - 14b &= -49 \\ & \Rightarrow a^2 + 14b = 0 & \Rightarrow a^2 + 14a &= -49 \\ & & \Rightarrow a^2 + 14a + 49 &= 0 \Rightarrow (a + 7)^2 = 0 \\ & & \Rightarrow a &= -7 & b &= 7 \\ & & a + b &= -7 + 7 = 0 \end{aligned}$$

(2)

$$x^2 + mx + 1 \xrightarrow{\Delta=0} m^2 - 4 < 0 \Rightarrow (m - 2)(m + 2) < 0$$

$$\Rightarrow \begin{matrix} -2 & 2 \\ + & - \\ - & + \end{matrix} \Rightarrow m \in (-2, 2)$$

(1)

(1) $x^2 + mx + 1 \rightarrow x=1$ $\Rightarrow 1 + m + 1 = 0 \Rightarrow m = -2$

$$\begin{cases} \Delta = 0 \\ x = \frac{-b}{a} = 1 \end{cases} \rightarrow m^2 - 4 = 0 \rightarrow m = \pm 2, x = \frac{-m}{1} = 1 \rightarrow m = -2 \text{ (I)}$$

(2) \cup (II) $\rightarrow -2 < m < 2$

$$\epsilon - \frac{1}{x^2} > 0 \Rightarrow \epsilon x^2 - 1 > 0 \Rightarrow (x-1)(x+1) > 0$$

$$\Rightarrow \begin{matrix} -\frac{1}{\epsilon} & \frac{1}{\epsilon} \\ +\phi & -\phi+ \end{matrix} \Rightarrow \mathbb{R} - \left(-\frac{1}{\epsilon}, \frac{1}{\epsilon}\right) \checkmark$$

$a > 0 \Rightarrow \Delta > 0$ I
 $\Delta = 0 \Rightarrow \epsilon m^2 - \epsilon m = 0 \Rightarrow \epsilon m(m-1) = 0 \Rightarrow m < 0$ II
 $\Delta < 0 \Rightarrow \epsilon m^2 - \epsilon m < 0 \Rightarrow \epsilon m(m-1) < 0 \Rightarrow \begin{matrix} 0 & 1 \\ +\phi & -\phi+ \end{matrix} \Rightarrow m \in (0, 1)$ III

$\Rightarrow \bigcap_{I \cap II \cap III} = m \in [0, 1]$

$$\frac{\epsilon x^2 - 1}{x-1} \Rightarrow x-1 \neq 0 \Rightarrow x \neq 1 \Rightarrow x \neq \frac{1}{\epsilon} \Rightarrow a = \frac{1}{\epsilon}$$

$$\epsilon_{x+k} = \epsilon_{x+1} \xrightarrow{x=\frac{1}{\epsilon}} \epsilon + k = \epsilon \Rightarrow k = 0 \quad a+k = \frac{1}{\epsilon} + 0 = \frac{1}{\epsilon}$$

$$\epsilon a^2 + a \epsilon = x + \epsilon \Rightarrow \epsilon a^2 + \epsilon a = \epsilon \Rightarrow a^2 + a = 1$$

$$a^2 + a - 1 = 0 \Rightarrow \frac{a+1}{a-1} < \frac{a+1}{a-1}$$

$$x = -\frac{\epsilon}{2} \Rightarrow -\epsilon a + \epsilon = -\epsilon + b \Rightarrow -\epsilon a - b = -\epsilon$$

$$\Rightarrow -\epsilon a + \epsilon = -\epsilon \Rightarrow -\epsilon a = -2\epsilon \Rightarrow a = 2$$

$$x = -1 \Rightarrow -a = \epsilon + b \Rightarrow b = -a - \epsilon$$

$$-\epsilon - (-\epsilon) = 0$$