

$$a^2 + 2a = a^2 - 4 \Rightarrow 2a = -4 \Rightarrow a = -2$$

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$$f(2) = 3 = \frac{2+a}{2-b} \Rightarrow 2+a = 12-2b$$

$$g(2) = 3 = 2+b \Rightarrow b = -1$$

$$\xrightarrow{b=-1} 2+a = 12+2 \Rightarrow a = 11$$

$$\Rightarrow f(1) = \frac{1+11}{2+1} = \frac{12}{3} = 4$$

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$$2x^2 + ax + b \neq 0 \begin{cases} x=-1 \rightarrow 2-a+b=0 \\ x=2 \rightarrow 8+2a+b=0 \end{cases} \Rightarrow 3a+2b=0 \Rightarrow a = -\frac{2}{3}b$$

$$\Rightarrow 2 + 6 + b = 0 \Rightarrow b = -8$$

$$\Rightarrow f(1) = \frac{2+1}{2-8} = \frac{3}{-6} = -\frac{1}{2}$$

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$$-x^2 + ax + b \neq 0 \xrightarrow{x=-1} -1-a+b=0 \Rightarrow b-a=1 \quad \textcircled{I}$$

$$(x+1)^2 = x^2 + 2x + 1 \xrightarrow{x=-1} -1 + 2(-1) + 1 = -1 \quad \textcircled{II}$$

$$\xrightarrow{\textcircled{I}, \textcircled{II}} b-a=1 \Rightarrow -1+1=0 \quad \checkmark$$

$$\Rightarrow a+b = 1-1 = 0$$

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$$x^2 + mx + 1 \xrightarrow{x=1} 1+m+1=0 \Rightarrow m = -2 \quad \textcircled{I}$$

$$\Delta < 0 \rightarrow m^2 - 4 < 0 \Rightarrow m^2 < 4$$

$$\Rightarrow -2 < m < 2 \quad \textcircled{II}$$

$$\xrightarrow{\textcircled{I}, \textcircled{II}} m = [-2, 2)$$

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$$x^r \neq 0 \Rightarrow x \neq 0 \text{ (I)}$$

$$r - \frac{1}{x^r} \geq 0 \Rightarrow \frac{-1}{x^r} \geq -r \Rightarrow \frac{1}{x^r} \leq r \Rightarrow 1 \leq r x^r \Rightarrow x^r \geq \frac{1}{r} \Rightarrow \frac{-1}{r} \leq x \leq \frac{1}{r} \text{ (II)}$$

$$\xrightarrow{\text{(I), (II)}} x = \left[ \frac{-1}{r}, \frac{1}{r} \right] - \{0\}$$

$$\Delta = f(m^r) - f(m) = r m^r - r m$$

$$\Delta \leq 0 \Rightarrow r m^r - r m \leq 0 \Rightarrow m^r - m \leq 0 \Rightarrow m(m-1) \leq 0$$

$$\rightarrow \frac{0}{+} \frac{1}{-} \Rightarrow m = [0, 1]$$

$$\frac{r x^r - 1}{r x - 1} = \frac{(r x - 1)(r x + 1)}{r x - 1} = r x + 1$$

$$r a - 1 = 0 \Rightarrow r a = 1 \Rightarrow a = \frac{1}{r} \text{ (I)}$$

$$r x + k \xrightarrow{x = \frac{1}{r}} r + k = r \Rightarrow k = 0 \text{ (II)}$$

$$r x + 1 \xrightarrow{x = \frac{1}{r}} = r$$

$$\xrightarrow{\text{(I), (II)}} a + k = \frac{1}{r} + 0 = \left[ \frac{1}{r} \right]$$

$$\frac{9 x^r - r}{r x + r} = \frac{(r x - r)(r x + r)}{r x + r} = r x - r = r x + b \Rightarrow b = -r \text{ (I)}$$

$$g\left(\frac{-r}{r}\right) = -r - r = -r = f\left(\frac{-r}{r}\right) = -r a + r \Rightarrow -r a + r = -r \Rightarrow -r a = -r \Rightarrow a = r \text{ (II)}$$

$$\xrightarrow{\text{(I), (II)}} a - b = r - (-r) = r + r = [2r]$$

$$f(x) = \frac{x^r - r}{x - r} = \frac{(x+r)(x-r)}{x-r} = x+r$$

$$f(r) = r a^r + r a = g(r) = r \Rightarrow r a^r + r a = r \Rightarrow a^r + a = 1$$

$$\Rightarrow a = [1, -r]$$