

$$f(x) = \begin{cases} x^2 + 2x & x > a \\ ax - 1 & x \leq a \end{cases} \quad \text{تابع پیوسته} \quad \begin{matrix} a < a \\ a > a \end{matrix} \quad \xrightarrow{a=a} \quad a^2 + 2a = a^2 - 1 \Rightarrow 2a + 1 = 0$$

$\Rightarrow a = -\frac{1}{2}$

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$$g(x) = 2x + b \quad (2, 3) \quad 3 = 4 + b \Rightarrow b = -1$$

$$f(x) = \frac{ax^2 + a}{2x - b} \quad (2, 3) \quad x = \frac{x+a}{x+1} \Rightarrow x+a = x+1 \Rightarrow a = 1$$

$$f(1) = \frac{1+1}{2-1} = 2$$

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$$f(x) = \frac{x^2 + 1}{2x^2 + ax + b} \quad D_f = \mathbb{R} - \{1, 2\} \Rightarrow \begin{cases} 2 - a + b = 0 \Rightarrow a - b = 2 \Rightarrow a = b + 2 \\ 4 + 2a + b = 0 \xrightarrow{a=b+2} 4 + 2(b+2) + b = 0 \\ \Rightarrow 4 + 2b + 2 + b = 0 \Rightarrow 3b = -6 \Rightarrow b = -2 \\ \Rightarrow a = 0 \end{cases}$$

$$f(1) = \frac{1+1}{2-4-2} = \frac{2}{-4} = -\frac{1}{2}$$

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$$f(x) = \frac{x^2 - \sqrt{x}}{-x^2 + ax + b} \quad D_f = \mathbb{R} - \{1\} \Rightarrow \begin{cases} 1 - a + b = 0 \Rightarrow a - b = 1 \\ f(1) = \frac{1 - 1}{-1 + a + b} = -1 \Rightarrow \frac{0}{-1 + a + b} = -1 \end{cases}$$

$a + b = -1 - 1 = -2$

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~~$f(x) = \frac{x^2 - 1}{x^2 + 1}$~~

~~$D_f = \mathbb{R} - \left\{ \frac{-1}{2}, \frac{1}{2} \right\}$~~

$$f(x) = \frac{2x}{(x-1)(x^2 + mx + 1)} \quad D_f = \mathbb{R} - \{1\}$$

$\begin{cases} \text{درباره } x^2 + mx + 1 \neq 0 \text{ در } x=1 \\ \Rightarrow \Delta < 0 \Rightarrow m^2 - 4 < 0 \\ \Rightarrow m \in (-2, 2) \text{ (1)} \\ \text{یا } x^2 + mx + 1 \text{ در } x=1 \text{ صفر باشد} \\ \Rightarrow m = -2 \text{ (2)} \end{cases}$

① \cup ② $\Rightarrow -2 < m < 2$

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$$f(u) = \sqrt{\frac{u-1}{a^2}}$$

$a^2 \neq 0 \rightarrow a \neq 0$
 $\frac{u-1}{a^2} > 0 \Rightarrow u > \frac{1}{a^2} \vee u < -\frac{1}{a^2}$

$$\Rightarrow D_f = \mathbb{R} - \left(-\frac{1}{a^2}, \frac{1}{a^2}\right)$$

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$$f(u) = \sqrt{m^2 + 2m + 1}$$

$m^2 + 2m + 1 > 0 \rightarrow m > -1$
 $\Rightarrow f_{m^2} - f_m \leq 0 \Rightarrow f_m(m-1) \leq 0$
 $\frac{0}{+b-b} \Rightarrow m \in [0, 1]$

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$$f(u) = \begin{cases} \frac{au^2-1}{a+1} & a \neq -1 \\ au+k & a = -\frac{1}{a} \end{cases} \quad g(u) = 2u+1$$

$a = -\frac{1}{a} \Rightarrow g(u) = 2 = f(u) = \frac{a^2+1}{a+1} + k \Rightarrow k = 0$
 $a = a \rightarrow a-1 = 0 \Rightarrow a = 1$
 $a+k = \frac{1}{a}$

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$$f(u) = \begin{cases} \frac{au^2-f}{a+1} & a \neq -\frac{1}{a} \\ au+1 & a = -\frac{1}{a} \end{cases} \quad g(u) = 2u+b$$

$a = 0 \Rightarrow \frac{a \cdot 0 - f}{a+1} = 2 \cdot 0 + b \Rightarrow \frac{-f}{1} = b \Rightarrow b = -f$
 $a = -\frac{1}{a} \Rightarrow 2 \cdot \left(-\frac{1}{a}\right) + 1 = 2 \cdot \left(-\frac{1}{a}\right) + b \Rightarrow -\frac{2}{a} + 1 = -\frac{2}{a} + b \Rightarrow b = 1$
 $a-b = 2 - (-f) = \frac{2}{a}$

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$$f(u) = \begin{cases} \frac{au^2-f}{a-1} & a \neq 1 \\ au+1 & a = 1 \end{cases} \quad g(u) = u+1$$

$a = 1 \rightarrow 1+1 = 1 \cdot a^2 + 1 \Rightarrow 2 = a^2 + 1 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$
 $\Rightarrow 1 \cdot (a+1)(a-1) = 0 \Rightarrow a = +1$
 $a = -1$

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