

الف) تابع است ✓ : $y^3 - 2x^2 = x^{n-1} \rightarrow y^3 = 2x^2 + x^{n-1} \rightarrow y = \sqrt[n]{2x^2 + x^{n-1}}$

ب) $x \cos y = y \cos x : \xrightarrow{n=\frac{\pi}{2}} \frac{\pi}{2} \times \cos y = y \times \cos \frac{\pi}{2} \rightarrow \frac{\pi}{2} \times \cos y = 0 \rightarrow \cos y = 0 \rightarrow y = \frac{\pi}{2} + \frac{\pi}{2}$
چنین می توان نوشت $\leftarrow$ تابع نیست x

ج) $\frac{x+y}{2} = \sqrt{xy} : \xrightarrow{(\cdot)^2} \frac{x^2 + y^2 + 2xy}{4} = xy \rightarrow x^2 + y^2 + 2xy = 4xy \rightarrow x^2 + y^2 - 2xy = 0 \rightarrow (x-y)^2 = 0 \rightarrow x=y$
تابع است ✓ (البته دارای $x=y$)

د) $xy^2 + 2x - y^2 + 2y = 0 : \rightarrow x(y^2 + 2) - y(y^2 + 2) = 0 \rightarrow (x-y)(y^2 + 2) = 0$
 $\xrightarrow{x \neq y} y^2 + 2 \neq 0$
تابع است $\leftarrow x=y$

الف) $|y| + x^2 = 2y \rightarrow x^2 = 2y - |y| \Rightarrow 2y - |y| \geq 0 \rightarrow 2y \geq |y| \rightarrow y \geq \frac{|y|}{2}$
 $\xrightarrow{y \geq 0} x^2 = 2y - y = y \rightarrow y = x^2$ تابع است ✓

ب) $[y] + [x] = 1 : \xrightarrow{x=0} [y] + 0 = 1 \rightarrow [y] = 1 \rightarrow 1 \leq y < 2$
چنین می توان نوشت $\leftarrow$ تابع نیست x

تابع الف درست است.

$f(n) = \begin{cases} 2n^2 - b & |n-1| > 2 \rightarrow n-1 > 2 \rightarrow n > 3 \\ a + 2n & \leq n \leq 3 \end{cases} \Rightarrow f(2) = \begin{cases} 2n^2 - b & n > 3 \\ a + 2n & \leq n \leq 3 \\ 2n^2 - b & n \leq -1 \end{cases}$

(نکته):
 $\xrightarrow{n=3} 2(3)^2 - b = a + 2(3) \rightarrow 18 - b = a + 6 \rightarrow \underline{a+b=12}$

$f(n) = \sqrt{a - bn - n^2} \rightarrow D_f = [b, a] \rightarrow a+b=?$

$\hookrightarrow a - bn - n^2 \geq 0 \Rightarrow n_1, n_2 = a, b \Rightarrow -n^2 - bn + a = 0 \Rightarrow (n-a)(n-b) = f(n)$

$-(n^2 - (a+b)n + ab) = f(n) \rightarrow -n^2 + (a+b)n - ab = \frac{-n^2 - bn + a}{f(n)} \rightarrow \begin{cases} a+b = -b \rightarrow a = -2b = -2(-1) \\ -ab = a \rightarrow \boxed{b=-1} \quad \boxed{a=2} \end{cases}$

$\Rightarrow a+b = 2-1 = \underline{1}$

$y = \frac{x}{\sqrt{an^2 + bn + c}} \rightarrow D_y = (-\infty, \infty) \rightarrow 3a + c - 3|b| = 9$ (بر حسب b)

فقط اینه پس $\Delta = 0$ اما در این صورت دو طرفه را به (۲) در جدول تعیین علامت

باید باشد که این عبارت درجه ۲ نبوده و $\Delta = 0$ $\leftarrow \frac{b^2}{4} - \frac{c}{a} = 0$

$bn + c \xrightarrow{n=0} 2b + c = 0 \rightarrow \boxed{c = -2b}$

$\hookrightarrow b \leq 0 \rightarrow$ بله جدول تعیین علامت

جواب: $3a^2 + c - 3|b| = -2b - 3|b| \rightarrow -2b - 3(-b) = -2b + 3b = \underline{b}$

$y = \sqrt{\frac{x^2 + 2}{x + b}} + a \rightarrow Dy = (x, +\infty) \rightarrow b = ?$	عبارت با ۲ ریشه یکی در صورت و یکی در مخرج داشته باشد : $\frac{0}{-1+} \frac{0}{+1-} \frac{0}{+1+}$ مشتق با این روش : $2x + 4 + 2 - 2a = 0$ بسیار ساده فقط ! $a + 2 = 0 \rightarrow a = -2$ $b = -2$
الف) $y = \sqrt{x^3 - x} : x^3 - x \geq 0 \rightarrow x(x^2 - 1) \geq 0$ ب) $y = \frac{1}{\sqrt{ x - 1}} : x - 1 > 0 \rightarrow x > 1 \rightarrow \begin{cases} x > 1 \\ x < -1 \end{cases} \rightarrow Dy = \mathbb{R} - [-1, 1] = (-\infty, -1) \cup (1, +\infty)$	الف) $y = \sqrt{x^3 - x} : x^3 - x \geq 0 \rightarrow x(x^2 - 1) \geq 0$ ب) $y = \frac{1}{\sqrt{ x - 1}} : x - 1 > 0 \rightarrow x > 1 \rightarrow \begin{cases} x > 1 \\ x < -1 \end{cases} \rightarrow Dy = \mathbb{R} - [-1, 1] = (-\infty, -1) \cup (1, +\infty)$
الف) $y = \sqrt{\frac{1}{ x - [2]}} : x - [2] \neq 0 \rightarrow x \neq [2] \rightarrow Dy = \mathbb{R} - \{2\} = \mathbb{R}^+$ ب) $\sqrt{x} + \sqrt{y} = 9 : \sqrt{y} = 9 - \sqrt{x} \rightarrow 9 - \sqrt{x} \geq 0 \rightarrow \sqrt{x} \leq 9 \rightarrow x \leq 81 \rightarrow 0 \leq x \leq 81$ $Dy = [0, 81]$	الف) $y = \sqrt{\frac{1}{ x - [2]}} : x - [2] \neq 0 \rightarrow x \neq [2] \rightarrow Dy = \mathbb{R} - \{2\} = \mathbb{R}^+$ ب) $\sqrt{x} + \sqrt{y} = 9 : \sqrt{y} = 9 - \sqrt{x} \rightarrow 9 - \sqrt{x} \geq 0 \rightarrow \sqrt{x} \leq 9 \rightarrow x \leq 81 \rightarrow 0 \leq x \leq 81$ $Dy = [0, 81]$
$y = \frac{1}{\sqrt{[x - \frac{1}{4}] + [x + \frac{3}{4}]}} : Dy = ? \rightarrow \text{مخرج} \neq 0 \rightarrow [x - \frac{1}{4}] + [x + \frac{3}{4}] > 0$ الف) $[x - \frac{1}{4} + 1 - 1] + [x + \frac{3}{4}] > 0 \rightarrow [x + \frac{3}{4}] - 1 + [x + \frac{3}{4}] > 0 \rightarrow 2[x + \frac{3}{4}] - 1 > 0$ ب) $2[x + \frac{3}{4}] > 1 \rightarrow [x + \frac{3}{4}] > \frac{1}{2} \rightarrow [x + \frac{3}{4}] \geq 1 \rightarrow x + \frac{3}{4} \geq 1 \rightarrow x \geq \frac{1}{4}$ $Dy = [\frac{1}{4}, +\infty)$	$y = \frac{1}{\sqrt{[x - \frac{1}{4}] + [x + \frac{3}{4}]}} : Dy = ? \rightarrow \text{مخرج} \neq 0 \rightarrow [x - \frac{1}{4}] + [x + \frac{3}{4}] > 0$ الف) $[x - \frac{1}{4} + 1 - 1] + [x + \frac{3}{4}] > 0 \rightarrow [x + \frac{3}{4}] - 1 + [x + \frac{3}{4}] > 0 \rightarrow 2[x + \frac{3}{4}] - 1 > 0$ ب) $2[x + \frac{3}{4}] > 1 \rightarrow [x + \frac{3}{4}] > \frac{1}{2} \rightarrow [x + \frac{3}{4}] \geq 1 \rightarrow x + \frac{3}{4} \geq 1 \rightarrow x \geq \frac{1}{4}$ $Dy = [\frac{1}{4}, +\infty)$
$y = \sqrt{1 - \log(x - a)} \rightarrow Dy = (b, c) \rightarrow c - b = ?$ الف) $x - a > 0 \rightarrow x > a \rightarrow x > \frac{a}{1}$ ب) $1 - \log(x - a) \geq 0 \rightarrow \log(x - a) \leq 1 \rightarrow x - a \leq 10 \rightarrow x \leq 10 + a \rightarrow x \leq \frac{10 + a}{1}$ $\Rightarrow x \in (\frac{a}{1}, \frac{10 + a}{1}) : Dy = (\frac{a}{1}, \frac{10 + a}{1}) \rightarrow (b, c) \rightarrow \begin{cases} b = \frac{a}{1} \\ c = \frac{10 + a}{1} \end{cases} \rightarrow c - b = 10$	$y = \sqrt{1 - \log(x - a)} \rightarrow Dy = (b, c) \rightarrow c - b = ?$ الف) $x - a > 0 \rightarrow x > a \rightarrow x > \frac{a}{1}$ ب) $1 - \log(x - a) \geq 0 \rightarrow \log(x - a) \leq 1 \rightarrow x - a \leq 10 \rightarrow x \leq 10 + a \rightarrow x \leq \frac{10 + a}{1}$ $\Rightarrow x \in (\frac{a}{1}, \frac{10 + a}{1}) : Dy = (\frac{a}{1}, \frac{10 + a}{1}) \rightarrow (b, c) \rightarrow \begin{cases} b = \frac{a}{1} \\ c = \frac{10 + a}{1} \end{cases} \rightarrow c - b = 10$