

سوال ۱۰

(A) داند و در جواب

۲۰

الف) $y'' - 2ay' = 2a - 1 \rightarrow y'' - 2ay' + 2a - 1 = 0$

$y' = 2a - 1 \rightarrow y' = y_1 \rightarrow y_1 = y_2$
 $y'' = 2a - 1 \rightarrow y'' = y_1' \rightarrow y_1' = y_2'$

✓ جواب است

① (۲)

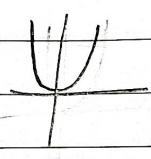
ب) $a \cos y = y \cos a \rightarrow a = \pi \rightarrow \cos \pi = -1 \rightarrow -1 = 1$ ✓
 $\cos \frac{\pi}{2} = 0$

ج) $\frac{u+y}{x} = \sqrt{xy} \rightarrow u+y = \sqrt{xy} \rightarrow (u+y)^2 = xy \rightarrow \sqrt{u} - \sqrt{y} = 0 \rightarrow \sqrt{u} = \sqrt{y} \rightarrow u = y$ ✓
 جواب است

د) $ay'' + 2a - y' - 2y = 0 \rightarrow a(y'' + 2) - y(y' + 2) = 0$ ✓
 $(y' + 2)(a - y) = 0 \rightarrow a = y$
 جواب است



ه) $|y| + a^2 = 2y \rightarrow y > 0 \rightarrow y = a^2$
 $y < 0 \rightarrow -y = a^2 \rightarrow y = -a^2$ ✓
 جواب است



② (۲)

و) $[y] + [a] = 1$
 $a = -1 \rightarrow [a] = 0 \rightarrow [y] = 1 \rightarrow y \in (1, 2]$ ✓
 جواب است

ز) $f(x) = \begin{cases} x^2 - b & |a-1| \geq 1 \\ a+x & 0 \leq x \leq 3 \end{cases} \quad a+b=?$
 $a-1 \geq 1 \rightarrow a \geq 2 \rightarrow x^2 - b$
 $-a+1 \geq -1 \rightarrow -1 \geq a \rightarrow x^2 - b$
 $a \leq -1$
 $f(x) = \begin{cases} x^2 - b & x > 3 \\ a+x & 0 \leq x \leq 3 \\ x^2 - b & x \leq -1 \end{cases} \quad a+b=?$
 $a=3 \rightarrow 1a-b=a+4 \rightarrow a+b=12$ ✓

ح) $f(x) = \sqrt{a-bx-ax^2} \quad D_f = [a, b] \quad a \neq 0 \quad b \in \mathbb{Z} \quad a+b=?$
 $S = a+b = -\frac{b}{a} = \frac{-(-b)}{-1} = -b \rightarrow a+b = -b \rightarrow a = -2b \rightarrow a = -2(-1) = +2$
 $D = ab = \frac{c}{a} = \frac{a}{-1} = -a \rightarrow ab = -a \rightarrow b = -1 \rightarrow a+b = 2-1 = 1$ ✓

ط) $y = \frac{x}{\sqrt{ax^2 + bx + c}} \quad D_f = (-\infty, 2) \quad 3a+c-3|b|$
 $3a+c-3|b| = -2b+3b = +b$ ✓
 $b \leq 0 \rightarrow -3|b| = +3b$

ث) $y = \sqrt{\frac{3ax}{a+b} + a} \quad (2, +\infty) \quad b=?$
 $\frac{3ax + r + a(a+b)}{a+b} = \frac{3ax + r + a^2 + ab}{a+b} = \frac{a(3x+a) + r + ab}{a+b}$
 $a+b=0 \rightarrow a = -b \rightarrow r+b=0 \rightarrow b = -r$ ✓

ii) $y = \sqrt{a^2 - x}$ $D_y = [-1, 0] \cup [1, +\infty)$ ✓

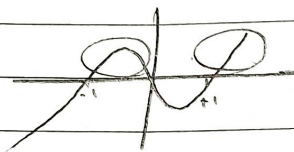
(r)

$a(a^2 - 1) \geq 0 \rightarrow a(a-1)(a+1) \geq 0$

$\Rightarrow y = \frac{1}{\sqrt{|x| - 1}}$

$\sqrt{|x| - 1} \geq 0 \rightarrow |x| - 1 \geq 0 \rightarrow |x| \geq 1 \rightarrow x \geq 1$
 $x \leq -1$

$D_y = (-\infty, -1) \cup (1, +\infty)$ ✓



iii) $y = \sqrt{\frac{1}{|x| - [a]}}$ $|a| \neq [a]$
 $D_y = \mathbb{R} - \mathbb{W}$ ✓

$\Rightarrow \sqrt{x} + \sqrt{y} = 9$ $\sqrt{x} \rightarrow a \geq -1$ (A)

$\sqrt{y} = 9 - \sqrt{x}$ $9 - \sqrt{x} \geq 0$

$9 \geq \sqrt{x} \rightarrow -1 \leq x \leq 81$ II (r)

$\sqrt{x} \leq 9$ $I \cap II \rightarrow D_y = [-1, 81]$ ✓

$y = \frac{1}{[x - \frac{1}{r}] + [x + \frac{r}{r}]}$

$[x - \frac{1}{r}] + [x + \frac{r}{r}] + 1 \geq 0$

$[x - \frac{1}{r}] \geq -1 \rightarrow [x - \frac{1}{r}] \geq -1$ (r)

$x - \frac{1}{r} \geq 0 \rightarrow x \geq \frac{1}{r} \rightarrow x \geq \frac{1}{r} \rightarrow D_y = [\frac{1}{r}, +\infty)$ ✓

(9)

$y = \sqrt{1 - \log(x - a)}$ $D_y = (b, c]$ $c - b = 9$

$x - a \geq 0 \rightarrow x \geq a \rightarrow a \geq \frac{9}{r}$

$1 - \log(x - a) \geq 0 \rightarrow \log(x - a) \leq 1$ (r)

$x - a \leq 10 \rightarrow x \leq \frac{10 + a}{r}$

$\frac{9}{r} \leq x \leq \frac{10 + a}{r}$

$c - b = \frac{10 + a}{r} - \frac{9}{r} = \frac{10 + a - 9}{r}$

$c - b = \frac{1}{r} = 9$ ✓

(10)