

دوازدهم تجربی

۲۰

هسته مرکزی

الف) $x^3 - 2x^2 = 2x - 1$

$$\begin{cases} y_1^c = 2x^2 + 2x - 1 \\ y_2^c = 2x^2 + 2x - 1 \end{cases}$$

$$\Rightarrow \{y_1^c = y_2^c \Rightarrow y_1 = y_2 \quad \boxed{\text{تابع است}} \quad \text{✓}$$

ب) $x \cos y = y \cos x \xrightarrow{x=y} \frac{\pi}{2} \cos y = \sin y$
 $\downarrow$
 $y = \frac{\pi}{2}, \frac{5\pi}{2}, \dots$ تابع نیست ✓

ج) $\frac{x+y}{2} = \sqrt{xy}$, $x, y > 0$.

$$x+y = 2\sqrt{xy} \rightarrow x+y-2\sqrt{xy} = 0 \rightarrow (\sqrt{x}-\sqrt{y})^2 = 0$$

$$\begin{cases} \sqrt{x} = \sqrt{y} \\ \sqrt{y} = \sqrt{x} \end{cases} \quad \sqrt{x} = \sqrt{y} \xrightarrow{\text{مربع}} |x| = |y| \xrightarrow{y>0} y = x \quad \boxed{\text{تابع است}} \quad \text{✓}$$

د) $xy^2 + 2x - y^2 - 2y = 0$

$$x(y^2 + 2) = y^2 + 2y \rightarrow x = \frac{y(y^2 + 2)}{y^2 + 2} \Rightarrow x = y \quad \boxed{\text{تابع است}} \quad \text{✓}$$

ه) $\frac{|y| + x^2}{2} = 2y \Rightarrow 2y \geq 0$

$$y + x^2 = 2y \rightarrow y = x^2 \quad \boxed{\text{تابع است}} \quad \text{✓}$$

و) $[y] + [2x] = 1$

$$\xrightarrow{x=0} [y] = 1 \rightarrow 1 < y < 2$$

$$\boxed{\text{تابع نیست}} \quad \text{✓}$$

$$f(n) = \begin{cases} rn^r - b & |n-1| \geq r \\ a + rn & 0 \leq n \leq c \end{cases} \rightarrow f(n) = \begin{cases} rn^r - b & n \geq c, n \leq -1 \\ a + rn & 0 \leq n \leq c \end{cases} \quad (r) \quad (r)$$

$$n = r \rightarrow \cancel{11-b=a+r} \rightarrow 11-b=a+r \rightarrow \boxed{a+b=11} \checkmark$$

$$f(n) = \sqrt{a-bn-n^r} \quad -n^r - bn + a \geq 0 \quad (r) \quad (r)$$

$$P_f = [b, a] \quad \frac{b \quad a}{-\phi + \phi -}$$

$$S = a + b = -b \rightarrow -a = rb \\ P = abc = -a \rightarrow \boxed{b = -1}$$

$$\xrightarrow{b=-1} a = rb \rightarrow a = -r \rightarrow \boxed{a = r} \\ a + b = +r - 1 = \boxed{1} \checkmark$$

$$y = \frac{r}{\sqrt{an^r + bn + c}}$$

$$D_y = (-\infty, r) \\ an^r + bn + c \geq 0$$

$$\frac{r}{+\phi -} \quad (r) \quad a=0 \\ \text{منقسم} \rightarrow b < 0$$

$$\rightarrow y = bn + c \xrightarrow{(r, 0)} 0 = rb + c \rightarrow c = -rb$$

$$ra + c - c/|b| \geq 0 \rightarrow r - rb - r(-b) = \boxed{b} \checkmark$$

$$y = \sqrt{\frac{rn+r}{n+b} + a}$$

$$D_y = (r, +\infty)$$

$$\frac{rn+r}{n+b} + a \geq 0$$

$$\frac{rn+r}{n+b} + a \geq 0 \rightarrow \frac{r(n+1)}{n+b} + a \geq 0$$

$$\rightarrow \frac{rn+r+an+ab}{n+b} \geq 0 \rightarrow \frac{(r+a)n + r+ab}{n+b} \geq 0$$

$$r+a=0 \rightarrow \boxed{a=-r}$$

$$n+b=r \rightarrow \boxed{b=-r} \checkmark$$

$$a) y = \sqrt{n^2 - 1}$$

$$n^2 - 1 \geq 0 \rightarrow n(n-1)(n+1) \geq 0$$

(r) (v)

$$D_f = [-1, 0] \cup [1, +\infty)$$

$$\begin{array}{c} -1 \quad 0 \quad 1 \\ - \quad + \quad - \quad + \end{array}$$

$$b) y = \frac{1}{\sqrt{|n| - 1}}$$

$$|n| - 1 \geq 0 \rightarrow |n| \geq 1$$

(v)

$$D_f = (-\infty, -1) \cup (1, +\infty)$$

$$c) y = \sqrt{\frac{1}{|n| - [n]}}$$

$$|n| - [n] = 0 \rightarrow |n| = [n] \xrightarrow[n \in \mathbb{Z}]{n \geq 0} n = n$$

(n)

$$D_f = \mathbb{R} - (\underbrace{\mathbb{Z}^+ \cup \{0\}}_{\text{nw}})$$

(r)

$$d) \sqrt{x} + \sqrt{y} = 9$$

$$x \geq 0 \quad (I)$$

(n)

$$y = -\sqrt{x} + 9$$

$$-\sqrt{x} + 9 \geq 0 \rightarrow \sqrt{x} \leq 9 \rightarrow |x| \leq 81$$

$$(I) \cap (II) \Rightarrow D_f = [0, 81]$$

(r) (q)

$$y = \frac{1}{[r_n - \frac{1}{r}] + [r_n + r - \frac{1}{r}]}$$

$$[r_n - \frac{1}{r}] + [r_n + r - \frac{1}{r}] > 0 \rightarrow r[r_n - \frac{1}{r}] > -r \rightarrow [r_n - \frac{1}{r}] > -1$$

$$\rightarrow 0 \leq r_n - \frac{1}{r} \rightarrow n \geq \frac{1}{2} \quad D_f = [\frac{1}{2}, +\infty)$$

$$y = \sqrt{1 - \log(rn - a)}$$

$$1 - \log(rn - a) \geq 0 \rightarrow rn - a \leq 1 \rightarrow rn \leq \frac{1+a}{r}$$

$$D_f = (b, c]$$

$$b = \frac{a}{r} \quad c = \frac{1+a}{r}$$

$$c - b = \frac{1+a}{r} - \frac{a}{r} = \frac{1}{r} \quad (2) \checkmark$$