

نام و نام خانوادگی: زهرا سادات حسینی | ریاضیات: تالیف سال ۱۳۹۵ | آدرس: برویه A دختر دوازده ساله

$$f(x) = x^3 - x + 1 \geq 0 \xrightarrow{f(x)} \begin{cases} x < 0 \rightarrow x^3 - x + 1 \geq 0 \rightarrow x^3 - x \geq -1 \rightarrow x^2 \geq -\frac{1}{x} \Rightarrow -\frac{1}{x} \leq x^2 < 0 \\ x \geq 0 \rightarrow x^3 - x \geq -1 \rightarrow x^3 - x \geq -1 \end{cases}$$

$$D = [-\frac{1}{x}, +\infty) : \cup$$

$$\frac{x^3 + 3x^2 + 18x + 3}{(x^2 + x + 1)^2} = \frac{(x+1)(x^2 + 15x + 3)}{(x+1)^2} \quad \left| \begin{array}{l} \frac{x^3 + 3x^2 + 18x + 3}{x^2 + x + 1} \quad \left| \frac{x+1}{x^2 + x + 1} \right. \\ \frac{x^3 + 3x^2 + 18x + 3}{x^2 + x + 1} \\ \hline 2x^2 + 17x + 3 \\ \hline 2x^2 + 2x + 2 \\ \hline 15x + 1 \end{array} \right.$$

$$\frac{x(x+1)(x+1)^2}{(x+1)^2} = \frac{3}{x+1} \quad \left| \begin{array}{l} D = \mathbb{R} - \{-1\} \\ (x+1) \neq 0 \rightarrow x \neq -1 \end{array} \right.$$

$$f(0) = \frac{-r}{-1} = r \rightarrow g(0) = r = 0 + c \Rightarrow \underline{c = r} \quad b + c = r + r = 2r$$

$$g(1) = 1 + r = r \xrightarrow{g(1) = f(1)} f(1) = \frac{a+r}{1+b} = r \rightarrow a+r = r+b \rightarrow \left\{ \begin{array}{l} 3b - a = -1 \\ b + a = r \end{array} \right.$$

$$g(-1) = -1 + r = 1 \xrightarrow{g(-1) = f(-1)} f(-1) = \frac{-a+r}{-1+b} = 1 \rightarrow -a+r = -1+b \rightarrow \left\{ \begin{array}{l} b + a = r \\ 4b = r \rightarrow b = \frac{1}{4}r \\ a = \frac{3}{4}r \end{array} \right.$$

$$f(x) = g(x) \rightarrow \frac{x+1}{x-c} = \frac{(x+1)(x-1)}{x^2 + ax + b} \rightsquigarrow \begin{cases} c = 1 \\ a = -1 \\ b = 1 \end{cases}$$

$$(x-1)^2 = x^2 - 2x + 1$$

$$a + b + c = -1 + 1 + 1 = 1$$

$$f(n+1) = (n+1-r)^2(n+1-1) = n(n-r)^2 = n^3 - 2rn^2 + r^2n$$

$$f(1-n) = (1-n-r)^2(1-n-1) = -n(-n-r)^2 = -n(n+r)^2 = -n^3 - 2rn^2 - r^2n$$

$$f(n+1) + f(1-n) = -2rn^2$$

$$1 - \sqrt{x} = -1 \rightarrow x = 1 \rightarrow \sqrt{x} = 1$$

$$1 - \sqrt{x} = 1 \rightarrow x = -1 \rightarrow \sqrt[3]{-1} + 1 = 1$$

$$f(-1) + f(1) = 1 + 1 = 2$$

$$f(x + \sqrt{x}) = x^2 + 2x$$

~~find~~

$$\sqrt{x} = t \rightarrow t^2 + t = 4 \rightarrow t^2 + t - 4 = 0 \rightarrow (t+3)(t-1) = 0$$

$$f(4) = f(t + \sqrt{t}) = (t)^2 + 2(t) = 2 \cdot 4 + 1 = 9$$

$$f(x-1) = (x-1)^2 + x-1, \quad f(x+1) = (1-x)^2 - x+1$$

$$f(x-1) + f(1-x) = (x-1)^2 + x-1 + (1-x)^2 - x+1 = 0$$

مربعین یکدیگر را حذف می‌کنیم

$$g = \sqrt{b - \sqrt{x}} + \sqrt{x} \rightarrow b - \sqrt{x} \geq 0 \rightarrow x \leq \frac{b^2}{4}$$

$$f = \sqrt{\sqrt{x} - \sqrt{a}} + \sqrt{x} - 1 \rightarrow \sqrt{x} - \sqrt{a} \geq 0 \rightarrow x \geq \frac{a^2}{4}$$

$$\left\{ \begin{array}{l} a = \frac{b^2}{4} \\ b = \frac{a^2}{4} \end{array} \right\} \Rightarrow x = \frac{b}{4} = \frac{a}{4} = 1$$

$$f - g = k \xrightarrow{x=1} \sqrt{1 - \sqrt{a}} + \sqrt{1} - 1 - (\sqrt{\sqrt{1} - \sqrt{a}} + \sqrt{1} - 1) = k \rightarrow \sqrt{1 - \sqrt{a}} - \sqrt{\sqrt{1} - \sqrt{a}} = k$$

$$\frac{a}{4} - \frac{b}{4} - k = \sqrt{x} - \sqrt{a} - \frac{b}{4} - k = \sqrt{x} - \sqrt{a} - \frac{b}{4} - k = 0$$

$$D_{f \circ g} = D_f \cap D_g \rightarrow D_g = [-1, +\infty), \quad D_f = (-\infty, m]$$

$$f(x) = \sqrt{x - m} + n \rightarrow x \geq m \Rightarrow m = -1$$

$$g(x) = \sqrt{\sqrt{x} + 1} \rightarrow \sqrt{x} + 1 \geq 0 \rightarrow x \geq -1$$

$$f(r) - g(r) = \sqrt{r} = \sqrt{r - (-1)} + n - \sqrt{\sqrt{r} + 1} = r + n + \sqrt{r} \rightarrow n = \sqrt{r} - r$$