

$$f(x) = \begin{cases} x-1 & x \geq 0 \rightarrow x-1 \geq 0 \rightarrow x \geq 1 \rightarrow \text{I} \\ x+3 & x < 0 \rightarrow x+3 \geq 0 \rightarrow x \geq -3 \rightarrow -3 \leq x < 0 \rightarrow \text{II} \end{cases}$$

$$f(x) \geq x-1 \rightarrow \text{I} \cup \text{II} \rightarrow \left[-\frac{1}{3}, 0\right) \cup [0, +\infty) \rightarrow \boxed{\left[-\frac{1}{3}, +\infty\right)}$$

$$y = \frac{3(2x+1)^3}{((2x+1)^2)^2} = \frac{3}{2x+1} \rightarrow 2x \neq -1 \rightarrow x \neq -\frac{1}{2}$$

$$D_y = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$x \neq \pm 1 \rightarrow f(x) = g(x) \rightarrow \frac{(x-1)(x+1)(x+2)}{(x-1)(x+1)} = x+C \rightarrow x+2 = x+C \rightarrow \boxed{C=2}$$

$$x \begin{cases} \xrightarrow{+1} \frac{a+2}{1+b} = 1+2 \rightarrow 2b-a = -1 \\ \xrightarrow{-1} \frac{-a+2}{-1+b} = 1 \rightarrow a+b = 2 \end{cases} \rightarrow \begin{cases} 2b-a = -1 \\ a+b = 2 \end{cases} \rightarrow b = \frac{1}{3} \rightarrow \boxed{b = \frac{1}{3}}$$

$$b+c = \frac{1}{3} + 2 = \frac{7}{3} \rightarrow \boxed{\frac{7}{3}}$$

$$f(x) = g(x) \rightarrow \frac{x+1}{x-c} = \frac{(x+1)(x-1)}{(x-1)(x-1)} \rightarrow \begin{cases} C=1 \\ b=1 \\ a=-2 \end{cases} \rightarrow \begin{cases} (x-1)^2 = x^2 - 2x + 1 \\ (x-1)^2 = x^2 - 2x + 1 \end{cases}$$

$$a+b+C = \boxed{0}$$

$$g(x) = (x-2)^2(x) - (x)(-x-2)^2 = x(x^2 - 4x + 4 - x^2 - 4x - 4) = x(-1x) = -1x^2 \Rightarrow \boxed{g(x) = -1x^2}$$

$$\begin{aligned} f(-1) &\Rightarrow 1 - \sqrt{x} = -1 \rightarrow x = 1 \xrightarrow{x=1} \sqrt{1} = 1 \\ f(3) &\Rightarrow 1 - \sqrt{x} = 3 \rightarrow x = -1 \xrightarrow{x=-1} \sqrt{-1} + 1 = 1 \end{aligned} \left. \vphantom{\begin{aligned} f(-1) \\ f(3) \end{aligned}} \right\} \rightarrow \sqrt{x+1} = \boxed{0}$$

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$$x + \sqrt{x} = 4 \rightarrow t^2 + t - 4 = 0 \rightarrow (t-2)(t+3) = 0 \rightarrow t = \sqrt{x} \begin{cases} \rightarrow -3 \rightarrow x = 9 \\ \rightarrow 2 \rightarrow x = 4 \end{cases}$$

$$f(4) = \sqrt{4} + 3(\sqrt{4}) = \boxed{10}$$

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$$f(x) = x(x^2 + 1)$$

$$\begin{aligned} f(x-1) + f(1-x) &= (x-1)((x-1)^2 + 1) - (x-1)((-x+1)^2 + 1) = \\ &= (x-1)(x^2 - 2x + 1 + 1 - x^2 + 2x - 1 - 1) = \boxed{0} \end{aligned}$$

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$$\begin{aligned} D_f &\rightarrow \sqrt{x-3} > 0 \rightarrow x > 3 \\ D_g &\rightarrow b - \sqrt{x} > 0 \rightarrow x < \frac{b^2}{4} \end{aligned} \left. \vphantom{\begin{aligned} D_f \\ D_g \end{aligned}} \right\} \xrightarrow{x=1} \boxed{a = \frac{4}{3}}, \boxed{b = 4}$$

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$$\begin{aligned} f-g &= \{ (1, K) \} \xrightarrow{x=1} \sqrt{1-3\left(\frac{K}{4}\right)} + K - 1 - \sqrt{1-K} - 1 - K = K \rightarrow \boxed{K = -1} \\ \psi\left(\frac{K}{4}\right) &= \frac{K}{4} - (-1) = \boxed{3} \end{aligned}$$

$$D_g = [-1, +\infty) \xrightarrow{m=V} D_f = (-\infty, V] \xrightarrow{x=V} m-V=0 \rightarrow \boxed{m=V}$$

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$$\sqrt{V-3} + n - 1\sqrt{V} = 4\sqrt{V} \rightarrow \boxed{n = 1\sqrt{V} - 2}$$

$$\boxed{m+n = 1\sqrt{V} + 5}$$