

$$f(x) = \begin{cases} x-1 & x \geq 1 \\ x+2 & x < 1 \end{cases}$$

$$\begin{aligned} f(x) \geq x-1 &\rightarrow x-1 \geq x-1 \quad x \geq 0 \quad \cap I \rightarrow x \geq 0 \\ &\rightarrow x+2 \geq x-1 \quad x \geq -\frac{1}{2} \quad \cap II \rightarrow x \geq -\frac{1}{2} \end{aligned} \quad \bigcup \quad x \geq -\frac{1}{2}$$

$$D = \left[-\frac{1}{2}, +\infty\right)$$

$$y = \frac{x^n (1 + x^{n+1} + x^{n+2} + \dots)}{(1 + x^{n+1})^2} = \frac{x^n (1 + x^{n+1})}{(1 + x^{n+1})^2} = \frac{x^n}{1 + x^{n+1}}$$

$x^{n+1} \neq 0 \quad x \neq -\frac{1}{x^n}$

$$D = \mathbb{R} - \left\{-\frac{1}{x^n}\right\}$$

$$f(x) = \begin{cases} \frac{x^n + x^{n+1} + \dots}{x^{n+1}} = \frac{x^n(n+1) + x^{n+1}(n+2) + \dots}{x^{n+1}} & x+1 > 0 \Rightarrow g(x) = x+C \Rightarrow C = \frac{1}{x} \\ \frac{x^n + x}{x+1} & \begin{cases} x \geq 1 \rightarrow \frac{x+1}{1+b} = g(1) = 1 & x+1 = 1+b \quad x = -b \\ x < -1 \rightarrow \frac{-x+1}{-1+b} = g(-1) = 1 & -x+1 = -1+b \quad -x-b = -2 \end{cases} \end{cases}$$

$$\Rightarrow -b = -2 \quad b = 2 \quad b + C = 2 + \frac{1}{x} = \frac{2x+1}{x}$$

$$f(x) = g(x) = \frac{x+1}{x-1} = \frac{x^2-1}{x^2+bx+c} = \frac{(x+1)(x-1)}{(x-1)(x-c)} \quad D_f = D_g$$

$$D_f = \mathbb{R} - \{1\} = \mathbb{R} - \{1, c\} \Rightarrow c = 1 \Rightarrow g(x) = \frac{x^2-1}{(x-1)^2} = \frac{x^2-1}{x^2-2x+1}$$

$$a = -2 \quad b = 1 \Rightarrow a + b + c = -2 + 1 + 1 = 0$$

$$g(x) = (x-2)^2 x + (1-x)^2 (-x) = (x-2)^2 x - (1-x)^2 x$$

$$= ((x-2)^2 - (1-x)^2) x = x(x-2+1-x)(x-2-1+x) = x \times (-1) \times (-1) = -1 \times x = -x$$

$$\begin{aligned} 1-2x &= -1 \quad x=1 \Rightarrow f(1) = 1 \\ 1-2x &= 1 \quad x=-1 \Rightarrow f(-1) = 1 \end{aligned} \quad \rightarrow \quad f(1) + f(-1) = 2$$

$$x + \sqrt{x} = y \quad \sqrt{x} = y - x \quad x = x^2 - 1^2 x + 1^2 y \quad x^2 - 1^2 x + 1^2 y = 0$$

$$(x-1)(x-y) = 0 \quad \rightarrow x=1 \text{ or } x=y$$

$$f(y) = f^2 + f \times f = 2xy + 1^2 = 2y + 1$$

$$f(x-1) = (x-1)^2 + (x-1) \quad f(1-x) = (1-x)^2 + (1-x)$$

$$(x-1)^2 + (x-1) - (1-x)^2 - (1-x) = 0$$

$$f(x) = \sqrt{x-a} + k-1$$

$$x \geq a$$

$$x \geq \frac{a}{f}$$

$$g(x) = \sqrt{b-x} + k$$

$$x \leq b$$

$$x \leq \frac{b}{f}$$

$$\frac{a}{f} = 1 \quad a = f \rightarrow f(x) = \sqrt{x-f} + k-1$$

$$\frac{b}{f} = 1 \quad b = f \rightarrow g(x) = \sqrt{f-x} + k$$

$$2f(1) - g(1) = 2k-2 - k = k \quad k = 1$$

$$a = \frac{b}{f} = k = f = 2+1 = 3$$

$$D_f = [-\infty, m]$$

$$D_g = [-1, +\infty]$$

$$\rightarrow D_f \cap D_g = [-1, \sqrt{2}] \Rightarrow m = \sqrt{2}$$

$$f(x) - g(x) = \sqrt{x-2} + k - \sqrt{2-x} = 2\sqrt{x} \quad x-2\sqrt{x}+k = 2\sqrt{x}$$

$$k = 2\sqrt{x} - x + 2$$

$$m = n = 2\sqrt{x} - x + 2 = 2\sqrt{2} + 2$$