

۲۰ امتحان عالی نوشتی

$$f(n) = \begin{cases} 2n-1 & : [n] > 4 \rightarrow n \geq 5 \\ 4n+3 & : [n] \leq 4 \rightarrow n \leq 4 \end{cases} \Rightarrow y = \sqrt{f(n)-n+1} \rightarrow f(n)-n+1 > 0$$

$$\rightarrow [n > 4] : 2n-1-n+1 \geq 0 \rightarrow [n \geq 5] \Rightarrow [n \geq 5]$$

$$\rightarrow [n \leq 4] : 4n+3-n+1 \geq 0 \rightarrow 3n+4 \geq 0 \rightarrow [n \geq -\frac{4}{3}] \Rightarrow [-\frac{4}{3} \leq n < 5] \Rightarrow D_y = [-\frac{4}{3}, +\infty)$$

$$y = \frac{2n^3 + 3n^2 + 12n + 3}{(4n^2 + 4n + 1)^2} = \frac{3 \times (1n^3 + 4n^2 + 4n + 1)}{(2n+1)^4} = \frac{3 \times (2n+1)^3}{(2n+1)^4} = \frac{3}{2n+1}$$

$$D_y \Rightarrow \text{مخرج} \neq 0 \rightarrow 2n+1 \neq 0 \rightarrow n \neq -\frac{1}{2} : D_y = \mathbb{R} - \{-\frac{1}{2}\}$$

$$f(n) = \begin{cases} \frac{n^2 + 2n^2 - n - 2}{n^2 - 1} & : n \neq \pm 1 \\ \frac{a+b}{n+b} & : n = \pm 1 \end{cases} \Rightarrow f(n) = g(n) : 1) [n=1] : \frac{a+2}{b+1} = 1+c$$

$$2) [n=-1] : \frac{-a+2}{b-1} = c-1$$

$$\Rightarrow a+2 = 1+b+c+bc \rightarrow a = b+c+bc-1$$

$$\hookrightarrow -a+2 = 1-b-c+bc \rightarrow a = b+c-bc+1 \quad \left. \begin{array}{l} b+c+bc-1 = b+c-bc+1 \\ 2bc = 2 \rightarrow bc = 1 \end{array} \right\} \rightarrow b+c = 1 + \frac{1}{b} \rightarrow b+c = 1 + \frac{1}{b}$$

$$f(n) = \frac{n+1}{n-c} \Rightarrow [n=0] : \frac{1}{-c} = \frac{x}{b} \Rightarrow [b=c]$$

$$g(n) = \frac{n^2-1}{n^2+an+b} \hookrightarrow D_f = D_g \hookrightarrow [n=1] : \frac{r}{1-c} = 0 \rightarrow r = 0$$

$$\Rightarrow f(n) : 1-c = - \rightarrow [c=1] \rightarrow [c=b=1] \quad \left. \begin{array}{l} a+b+c = -2+1+1 = 0 \\ \Rightarrow g(n) : 1+a+b = 0 \rightarrow [a=-2] \end{array} \right\} a+b+c = -2+1+1 = 0$$

$$f(n) = (n-3)^2(n-1) \Rightarrow g(n) = f(n+1) + f(1-n) : g(n) = (n+1-3)^2(n+1-1) + (1-n-3)^2(1-n-1)$$

$$\Rightarrow g(n) = (n-2)^2 \times n + (n+2)^2 \times -n \rightarrow g(n) = n(n^2 - 4n + 4 - n^2 - 4n - 4) = -1n^2$$

$$\Rightarrow g(n) = -1n^2$$

$$f(1-x) = \begin{cases} \sqrt{x} + x & : x \leq 0 \\ \sqrt{x} & : x > 0 \end{cases} \rightarrow f(-1) + f(x) = ? : 1-x = -1 \rightarrow -x = -2 \rightarrow \underline{x = -1}$$

$$\Rightarrow \underline{x = 1} : \sqrt{1} = 1 \checkmark : f(-1) / 1-x = 2 \rightarrow -x = 2 \rightarrow \underline{x = -1}$$

$$\Rightarrow \underline{x = -1} : \sqrt{-1} + x = 1 \checkmark : f(x) \Rightarrow f(-1) + f(x) = 1 + 1 = 2 \checkmark$$

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$$f(n+\sqrt{n}) = n^x + \sqrt{n} \rightarrow f(x) = ? : n+\sqrt{n} = x : \sqrt{n} = x-n \rightarrow -x = \sqrt{n} \text{ (impossible)} \rightarrow x = \sqrt{n} \rightarrow \underline{n = x^2}$$

$$\Rightarrow (x) + \sqrt{x} = x^2 + 1 \checkmark = \underline{x^2 + 1} \checkmark$$

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$$f(x) = x^3 + x \rightarrow f(x-1) + f(1-x) \stackrel{(\text{sol})}{=} (x-1)(x^2-x+1) + (1-x)(x^2-x+1) = (x^2-x+1)(x-x+x-x) = 0$$

$$\hookrightarrow x(x^2+1)$$

$$\underline{b} : \text{sol} : (x-1)^3 + (x-1) + (1-x)^3 + (1-x) = x^3 - 3x^2 + 3x - 1 + x - 1 + 1 - 3x^2 + 3x - 1 + 1 - x + x^3 = 2x^3 - 6x^2 + 6x - 2 = 2(x^3 - 3x^2 + 3x - 1) = 2(x-1)^3 \checkmark$$

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$$f(x) = \sqrt{x-a} + k-1 \rightarrow x-a \geq 0 \rightarrow x \geq \frac{a}{2} \Rightarrow 2f-g = \{(1, k)\}$$

$$g(x) = \sqrt{b-x} + k \rightarrow b-x \geq 0 \rightarrow x \leq \frac{b}{2} \} \rightarrow \left[\frac{a}{2}, \frac{b}{2} \right] \rightarrow \left\{ \begin{array}{l} \frac{a}{2} = 1 \rightarrow a = 2 \checkmark \\ \frac{b}{2} = 1 \rightarrow b = 2 \checkmark \end{array} \right.$$

$$\hookrightarrow 2f(1) - g(1) = k : 2(\sqrt{1-a} + k-1) - (\sqrt{b-1} + k) = k \rightarrow 2k-2-2\sqrt{1-a} - \sqrt{b-1} - k = k \rightarrow 2k-2-2\sqrt{1-a} - \sqrt{b-1} = k$$

$$\Rightarrow -k-2 = k \rightarrow 2k = -2 \rightarrow \underline{k = -1} \checkmark : a - \frac{b}{2} - k = ? : 2 - \frac{2}{2} - (-1) = 1 - (-1) = 2 \checkmark$$

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$$f(x) = \sqrt{m-x} + x \rightarrow D_{fg} = [-1, \sqrt{m}] \Rightarrow m-x \geq 0 \rightarrow x \leq m$$

$$g(x) = \sqrt{x+1} \rightarrow x+1 \geq 0 \rightarrow \underline{x \geq -1} \rightarrow \underline{m = \sqrt{m}} \checkmark$$

$$\Rightarrow f-g(x) = \sqrt{m-x} + x - \sqrt{x+1} = x + m - x\sqrt{x} = x\sqrt{x} \rightarrow \underline{x = \sqrt{x}} \checkmark$$

$$m+x = \sqrt{x} + 1 \checkmark$$

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