

$$f(1-x) = \begin{cases} \sqrt{x} + 1 & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases}$$

$$f(-1) + f(1) = f+1 = \omega$$

$$1-x = -1 \rightarrow -x = -2 \rightarrow x = 2 \quad f(-1) = f$$

$$1-x = 1 \rightarrow -x = 0 \rightarrow x = 0 \quad f(1) = 1$$

$$f(x+\sqrt{x}) = x^f + \sqrt{x}$$

$$f(y)$$

$$x+\sqrt{x} = y \rightarrow x = \sqrt{y} \rightarrow \sqrt{y} + \sqrt{\sqrt{y}} = y$$

$$f(y) = f + 1\sqrt{y} = \sqrt{y} \wedge$$

$$f(x) = x'' + x$$

$$f(x-1) + f(1-x) = 0$$

$$\left. \begin{aligned} f(x-1) &= (x-1)'' + x-1 = x'' - 2x' + 2x - 1 + x-1 \\ f(1-x) &= (1-x)'' + 1-x = 1 - 2x' + 2x - 1 + 1-x \end{aligned} \right\} \rightarrow 0$$

$$f(x) = \sqrt{\varepsilon x - \varepsilon a} + k-1 \quad g(x) = \sqrt{b - \varepsilon x} + \sqrt{k} \quad f-g = \{(1, k)\}$$

$$\left. \begin{aligned} \varepsilon a - \frac{b}{\varepsilon} - k &\Rightarrow \psi\left(\frac{\varepsilon}{\sqrt{k}}\right) - \frac{\varepsilon k}{\sqrt{k}} - k = f - \sqrt{k} \\ \sqrt{\varepsilon x - \varepsilon a} + \sqrt{k} - \sqrt{b - \varepsilon x} - \sqrt{k} &= \sqrt{\varepsilon x - \varepsilon a} - \sqrt{b - \varepsilon x} - k - 1 \end{aligned} \right\} \rightarrow x=1, y=k \rightarrow \mu = \sqrt{k} - \sqrt{k} - k - 1 \rightarrow k = -1$$

$$D = \left\{ \begin{aligned} \varepsilon x - \varepsilon a \geq 0 &\rightarrow \varepsilon x \geq \varepsilon a \rightarrow x \geq \frac{\varepsilon a}{\varepsilon} \\ b - \varepsilon x \geq 0 &\rightarrow b \geq \varepsilon x \rightarrow \frac{b}{\varepsilon} \geq x \end{aligned} \right\} \Rightarrow D = \left(\frac{\varepsilon a}{\varepsilon}, \frac{b}{\varepsilon} \right) \Leftrightarrow (1, k) \rightarrow \frac{1 - \frac{\varepsilon a}{\varepsilon}}{\varepsilon} \rightarrow a = \frac{\varepsilon}{\sqrt{k}} \rightarrow \frac{b}{\varepsilon} = k \rightarrow b = \varepsilon k$$

$$f(x) = \sqrt{m-x} + n$$

$$g(x) = \sqrt{2x+2}$$

$$D_{f \wedge g} = [-1, 4]$$

$$f-g(x) = 4\sqrt{2}$$

$$m+n=?$$

$$Dg \rightarrow 2x+2 \geq 0 \rightarrow x \geq -1$$

$$m-x \geq 0 \rightarrow m \geq x$$

$$\text{بأنه يجب أن يكون}$$

$$m=4$$

$$g(x) = \sqrt{2} = \sqrt{2}\sqrt{2}$$

$$f-g(x) = 4\sqrt{2} \rightarrow f = 4\sqrt{2}$$

$$f(x) = \sqrt{4-x} + n = 4\sqrt{2} \rightarrow n = 4\sqrt{2} - 2$$

$$m+n = 4\sqrt{2} - 2 + 4 = 4\sqrt{2} + 2$$

نکته: اگر $f(x)$ در $x=c$ پیوسته باشد و $g(x) = f(x) + f(1-x)$ باشد، آنگاه $g(x)$ در $x=\frac{1}{2}$ پیوسته است.

آفرین

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$$g = \sqrt{f(x) - x + 1}$$

$$f(x) = \begin{cases} rx-1 & [x] > \varepsilon \rightarrow x > \varepsilon \\ \varepsilon x + r & [x] \leq \varepsilon \rightarrow x \leq \varepsilon \end{cases}$$

$$x > \varepsilon \sim rx-1-x+1 \geq 0 \sim rx \geq 0 \sim x \geq 0 \Rightarrow x \geq \varepsilon$$

$$x < \varepsilon \sim \varepsilon x + r - x + 1 \geq 0 \sim rx + \varepsilon \geq 0 \sim rx \geq -\varepsilon \sim x \geq -\frac{\varepsilon}{r} \rightarrow x \in [-\frac{\varepsilon}{r}, \varepsilon)$$

$$D = [-\frac{\varepsilon}{r}, \infty)$$

$$g' = \frac{r(\varepsilon x^2 + rx + 1) - (rx-1)}{(\varepsilon x^2 + \varepsilon x + 1)^2} = \frac{r(\varepsilon x^2 + rx + 1 - rx + 1)}{(\varepsilon x^2 + \varepsilon x + 1)^2} = \frac{r(2\varepsilon x + 2)}{(\varepsilon x^2 + \varepsilon x + 1)^2} = \frac{2r(\varepsilon x + 1)}{(\varepsilon x^2 + \varepsilon x + 1)^2}$$

$$= \frac{2r}{(\varepsilon x + 1)^2}$$

$$D = R - \{-\frac{1}{r}\}$$

$$f(x) = \begin{cases} \frac{x^2 + rx^2 - x - r}{x^2 - 1} & x \neq \pm 1 \\ \frac{ax + r}{x + b} & x = \pm 1 \end{cases} \quad g(x) = x + c$$

$$\frac{x^2 + rx^2 - x - r}{x^2 - 1} = \frac{(x+r)(x^2-1)}{x^2-1} = x + c \Rightarrow c = r$$

$$\begin{aligned} x=1 & \Rightarrow \frac{a+r}{1+b} = r \Rightarrow a+r = r+b \\ x=-1 & \Rightarrow \frac{-a+r}{-1+b} = 1 \Rightarrow -a+r = -1+b \end{aligned} \Rightarrow \begin{cases} a+r = r+b \\ -a+r = -1+b \end{cases} \Rightarrow \begin{cases} a = b \\ r = -1+b \end{cases} \Rightarrow b = \frac{r+1}{2}$$

$$f(x) = \frac{x+1}{x-c} \quad g(x) = \frac{x^2-1}{x^2+ax+b} = \frac{(x+1)(x-1)}{(x^2+ax+b)} = \frac{(x+1)(x-1)}{(x-c)(x-1)}$$

$$\rightarrow x^2 - (c+1)x + c = x^2 + ax + b \Rightarrow a = -c-1, b = c$$

$$a+b+c = -c-1+c+c = c-1 \rightarrow 0$$

$$F(x) = (x-r)^2(x-1) \quad g(x) = F(x+1) + f(1-x)$$

$$g(x) = (x+1-r)^2(x+1-1) + (1-x-r)^2(1-x-1) = (x-r)^2(x) + (-x-r)^2(-x)$$

$$= (x-r)^2(x) - (-x-r)^2(x) = x[(x-r)^2 - (-x-r)^2] = x[x^2 - 2rx + r^2 - (x^2 + 2rx + r^2)] = x[-4rx] = -4rx^2$$

$$g(x) = -4rx^2$$