

$y \in \sqrt{f(x)-x+1}$, $f(x) = \begin{cases} x+1, (x) \in \mathbb{R} \rightarrow x \geq 0 \\ x+3, (x) \leq 0 \rightarrow x < 0 \end{cases}$

① $x \geq 0 \rightarrow x+1-x+1 \geq 0 \rightarrow x \geq 0 \rightarrow x \geq 0$
 $x < 0 \rightarrow x+3-x+1 \geq 0 \rightarrow x \geq -4 \rightarrow -4 \leq x < 0$
 $\Rightarrow D_y \in [-4, +\infty)$

$y = \frac{x^2 + 3x + 1}{(x^2 + x + 1)^2} \rightarrow x^2 + x + 1 \leq 0 \rightarrow x \leq -\frac{1}{2}$
 $D_y \in \mathbb{R} - \left\{ \frac{-1}{2} \right\} \Rightarrow D_y \in \mathbb{R} - \left\{ -\frac{1}{2} \right\}$

$y = \frac{(x+3)(x^2+x+1)}{(x^2+x+1)^2} = \frac{x(x+1)}{(x+1)^2} = \frac{x}{x+1}$

$g(x) \in x+c$, $f(x) = \begin{cases} \frac{x^2+x-2}{x-1} , x \neq 1 \\ \frac{a+b}{x+b} , x = 1 \end{cases}$

④ شرط برابری دو تابع : ① برابر بودن دامنی و متابع
 ② $f(x) = g(x) \Rightarrow$ از دو طرف

$x=0 \rightarrow g(0) \leq c \rightarrow g(0) = f(0) \Rightarrow (c \leq 2)$

$x=1 \rightarrow g(1) \leq 1+c \rightarrow g(1) = f(1) \Rightarrow \frac{a+b}{1+b} \leq 1+c \Rightarrow a+b \leq 1+c \Rightarrow a-b \leq 1$

$x=-1 \rightarrow g(-1) \leq 1 \rightarrow g(-1) = f(-1) \Rightarrow \frac{-a+b}{b-1} \leq 1 \Rightarrow -a+b \leq b-1 \Rightarrow -a \leq -1 \Rightarrow a \geq 1$

$b+c \leq \frac{1}{2} + c \Rightarrow \frac{a}{2}$

$f(x) \leq \frac{x+1}{x-c}$, $g(x) = \frac{x^2-1}{x^2+a+b}$

① $D_f \subseteq D_g \Rightarrow D_f \subseteq \mathbb{R} - \{c\} \subseteq D_g$
 $x^2+a+b \leq (x-c)^2 \Rightarrow x^2+a+b \leq x^2-2cx+c^2 \Rightarrow a+b \leq c^2-2cx+c^2$
 $f(x) \neq g(x)$
 $c \leq 0 \rightarrow \frac{x+1}{x} \neq \frac{x^2-1}{x^2} \Rightarrow x \neq \frac{1}{c} \Rightarrow \frac{1}{c} \neq \frac{1}{c}$
 $c > 0 \rightarrow \frac{x+1}{x-1} \leq \frac{(x+1)(x-1)}{(x-1)^2} \Rightarrow \frac{x+1}{x-1} \leq \frac{x^2-1}{(x-1)^2} \Rightarrow \frac{x+1}{x-1} \leq \frac{x+1}{x-1} \Rightarrow c \leq 1$

② $f(x) \leq g(x) \Rightarrow \frac{x+1}{x-c} \leq \frac{x^2-1}{x^2+a+b} \Rightarrow (x+1)(x^2+a+b) \leq (x^2-1)(x-c)$
 $\frac{1}{c} \leq \frac{1}{c^2} \Rightarrow c \leq c^2 \Rightarrow c^2 - c \geq 0 \Rightarrow c(c-1) \geq 0 \Rightarrow c \leq 0 \text{ or } c \geq 1$

$a+b+c \leq \frac{x^2}{-2} + \frac{c^2}{1} + \frac{c^2}{1} \Rightarrow 0$

$g(x) \leq f(x+1) + f(1-x)$

$f(x) = (x-3)^2(x-1)$
 $f(x+1) = (x-2)^2(x)$
 $f(1-x) = (-x-2)^2(-x-1)$
 $g(x) = x^3 + (x-2)^2x - x^3 - (-x-2)^2(-x-1) = -4x^2$

$f(-1) + f(3) \leq f$, $f(1-x) \leq \begin{cases} \sqrt{x+2} & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases}$

$1-2x \leq -1 \rightarrow x \geq 1 \Rightarrow f(1) \leq \sqrt{1+2} \leq f$
 $1-2x \leq 3 \rightarrow x \geq -1 \Rightarrow f(4) \leq \sqrt{-1+2} \leq -1+2 \leq 1 \Rightarrow f(-1) + f(3) \leq f+1 \leq f$

$$f(n+\sqrt{n}) \leq n^2 + 2\sqrt{n}, f(4) \leq ?$$

(7)

$$n+\sqrt{n} \leq 4 \xrightarrow{\sqrt{n} \leq 4-t} t^2+t-4 \leq 0 \quad \begin{cases} \sqrt{n} \leq -1/2 \\ \sqrt{n} \leq 4 \end{cases} \rightarrow n \leq 16 \Rightarrow f(4) \leq \frac{(4)^2}{209} + \frac{2(4)}{12} = 1.61$$

$$f(n) \leq n^2 + n, f(n-1) + f(1-n) \leq ?$$

(9)

$$f(n-1) \leq (n-1)((n-1)^2+1) \Rightarrow (n-1)((n-1)^2+1) - (n-1)((n-1)^2+1) \leq 0$$

$$f(1-n) \leq (1-n)((1-n)^2+1) \xrightarrow{(1-n)^2 \leq (n-1)^2} - (n-1)((n-1)^2+1)$$

$$f(n) \leq \sqrt{f(n-a)+k-1}, g(n) \leq \sqrt{b-f(n)+k}$$

(10)

$$f-g \leq \{1, k\} \Rightarrow D_f \cap D_g \leq \{1\}$$

$$f(1) - g(1) \leq k \Rightarrow 1 - k - 1 - k \leq k \Rightarrow k \leq -1$$

$$\begin{aligned} & \begin{cases} f(n-a) \geq 0 \\ f(n) \geq a \end{cases} \quad \begin{cases} b-f(n) \geq 0 \\ b \geq f(n) \end{cases} \\ & \left(n \geq \frac{f(n)}{a} \right) \wedge \left(n \leq \frac{b}{f(n)} \right) \Rightarrow \frac{fa}{f} \leq \frac{b}{f} \leq 1 \Rightarrow \left(\frac{b}{f} \leq 1 \right) \end{aligned}$$

$$\begin{aligned} & f(1) \leq \sqrt{f(1)-f} + k-1 \\ & g(1) \leq \sqrt{f-f(1)} + k \end{aligned}$$

$$fa - \frac{b}{f} - k \leq f-1+1 \leq 1$$

$$f(n) \leq \sqrt{m-n} + h, g(n) \leq \sqrt{r+n}, D_{f \cap g} = D_f \cap D_g \leq [-1, v]$$

(11)

$$f(1) - g(1) \leq 4\sqrt{r} \quad \begin{cases} f(1) \leq \sqrt{v-1} + h \leq 1+h \\ g(1) \leq \sqrt{r(1)+r} \leq \sqrt{r} \end{cases}$$

$$\begin{aligned} & m-n \geq 0 \quad \begin{cases} m \geq n \\ n \geq -1 \end{cases} \Rightarrow -1 \leq n \leq v \Rightarrow m \leq v \\ & r+n \geq 0 \quad \begin{cases} n \geq -r \\ n \leq v \end{cases} \Rightarrow m+n \leq v + \sqrt{r} \end{aligned}$$

$$r+h + \sqrt{r} \leq 4\sqrt{r} \Rightarrow n \leq \sqrt{r} - r \rightarrow m+n \leq v + \sqrt{r}$$