

$$f(x) = \begin{cases} 2x-1 & [x] > 4 \rightarrow x \geq 5 \quad (1) \\ 4x+3 & [x] \leq 4 \rightarrow x < 5 \quad (2) \end{cases} \quad y = \sqrt{f(x)-x+1}$$

سوال (۱)

$$Dy = \left[-\frac{4}{3}, 5\right) \cup [5, +\infty) = \left[-\frac{4}{3}, +\infty\right)$$

$$\begin{aligned} (1) & y = \sqrt{2x-1-x+1} \Rightarrow y = \sqrt{x} \rightarrow x \geq 0 \rightarrow [0, +\infty) \cap [5, +\infty) = [5, +\infty) \\ (2) & y = \sqrt{4x+3-x+1} \Rightarrow y = \sqrt{3x+4} \quad x \geq -\frac{4}{3} \rightarrow \left[-\frac{4}{3}, +\infty\right) \cap (-\infty, 5) = \left[-\frac{4}{3}, 5\right) \end{aligned}$$

$$\frac{25x^3 + 34x^2 + 14x + 3}{(5x^2 + 6x + 1)^2} = \frac{3(12x^3 + 12x^2 + 4x + 1)}{(2x+1)^4} = \frac{3(2x+1)^3}{(2x+1)^4} = \frac{3}{2x+1}$$

سوال (۲)

$$D_f = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$f(x) = \begin{cases} \frac{x^2 + 2x^2 - x - 2}{x^2 - 1} & x \neq \pm 1 \\ \frac{ax + 2}{x + b} & x = \pm 1 \end{cases}$$

سوال (۳)

$$\frac{x^2 + 2x^2 - x - 2}{x^2 - 1} = \frac{x^2(x+2) - (x+2)}{x^2 - 1} = \frac{(x+2)(x^2 - 1)}{x^2 - 1} = x+2 \quad \begin{matrix} x=1 \rightarrow 3 \\ x=-1 \rightarrow 1 \end{matrix}$$

$$(1, 3) \rightarrow \frac{a(1) + 2}{1 + b} = 3 \rightarrow a + 2 = 3b + 3 \rightarrow$$

$$(-1, 1) \rightarrow \frac{-a + 2}{-1 + b} = 1 \rightarrow -a + 2 = -1 + b \rightarrow$$

$$\begin{cases} 3b - a = -1 \\ a + b = 3 \\ 4b = 2 \rightarrow b = \frac{1}{2} \\ a = \frac{5}{2} \end{cases}$$

$$g(x) = x + c = x + 2 \rightarrow c = 2$$

$$b + c = \frac{1}{2} + 2 = \frac{5}{2}$$

سوال (۴)

$$g(x) = \frac{x^2 - 1}{x^2 + ax + b} = \frac{(x+1)(x-1)}{(x-1)(x^2 - 2x + 1)} = \frac{x+1}{x^2 - 1}$$

$$a + b + c = -2 + 1 + 1 = 0$$

$$f(x) = \frac{x+1}{x-c} = \frac{x+1}{x-1} \rightarrow c = 1$$

$$f(x) = (x-2)^2(x-1)$$

$$g(x) = f(x+1) + f(1-x) = x^3 - 6x^2 + 8x - 1 - x^3 - 6x^2 - 8x - 1 = -12x^2 \quad \text{سوال (۵)}$$

$$x+1 \rightarrow (x+1-2)^2(x+1-1) = (x-1)^2(x) = x^3 - 2x^2 + x$$

$$1-x \rightarrow (1-x-2)^2(1-x-1) = (-x-1)^2(-x) = -x^3 - 2x^2 - x$$

$$f(-1) \rightarrow 1-2x = -1 \rightarrow x=1 > 0 \rightarrow \sqrt[3]{x} \rightarrow \sqrt[3]{1} = 1$$

$$f(3) \rightarrow 1-2x = 3 \rightarrow x = -1 < 0 \rightarrow \sqrt[3]{x} + 2 \rightarrow \sqrt[3]{-1} + 2 = 1$$

$$f(-1) + f(3) = 1 + 1 = 2$$

$$f(x + \sqrt{x}) = x^2 + 2x$$

$$f(4) \rightarrow x + \sqrt{x} = 4 \rightarrow t^2 + t - 4 = 0 \rightarrow (t+3)(t-2) = 0 \rightarrow t = -3 \text{ یا } 2 \rightarrow \sqrt{x} = 2 \rightarrow x = 4$$

$$\rightarrow (4)^2 + 2(4) = 24$$

$$f(x-1) \rightarrow (x-1)^3 + (x-1) = x^3 - 3x^2 + 3x - 1 + x - 1 = x^3 - 3x^2 + 4x - 2$$

$$f(1-x) \rightarrow (1-x)^3 + (1-x) = 1 - 3x + 3x^2 - x^3 + 1 - x = -x^3 + 3x^2 - 4x + 2$$

$$f(x-1) + f(1-x) = x^3 - 3x^2 + 4x - 2 - x^3 + 3x^2 - 4x + 2 = 0$$

$$f(x) = \sqrt{4x - 3a} + k - 1$$

$$g(x) = \sqrt{b - 4x} + 3k$$

$$D_f = [\frac{3a}{4}, +\infty) \quad D_g = (-\infty, \frac{b}{4}] \rightarrow D_f \cap D_g = \{1\} \rightarrow \frac{3a}{4} = \frac{b}{4} \rightarrow 3a = b$$

$$2f(1) - g(1) = k \rightarrow 2(\sqrt{4 - 3a} + k - 1) - \sqrt{b - 4} - 3k = k \Rightarrow 2\sqrt{4 - 3a} + 2k - 2 - \sqrt{b - 4} - 3k = k$$

$$\xrightarrow{b=3a} 2\sqrt{4 - 3a} - k - 2 - \sqrt{3a - 4} = k \rightarrow k = 1$$

$$a \geq \frac{4}{3} > a \leq \frac{4}{3} \rightarrow a = \frac{4}{3} \quad b = 3a = 4 \quad 3a - \frac{b}{3} - k = 4 - 2 + 1 = 3$$

$$\left. \begin{aligned} g(x) &= \sqrt{2x+2} \rightarrow D_g = [-1, +\infty) \\ f(x) &= \sqrt{m-x} + n \rightarrow D_f = (-\infty, m] \end{aligned} \right\} \rightarrow D_f \cap D_g = [-1, m] \rightarrow D_f = (-\infty, m] \rightarrow m = 7$$

$$f(3) - g(3) = 4\sqrt{2} \rightarrow 3+n - \sqrt{2 \cdot 3 + 2} = 4\sqrt{2} \rightarrow 3+n = 8\sqrt{2} \rightarrow n = 8\sqrt{2} - 3$$

$$g(3) = \sqrt{8} = 2\sqrt{2}$$

$$m+n = 8\sqrt{2} - 3 + 7 = 8\sqrt{2} + 4$$

$$f(3) = \sqrt{m} + n = 2+n$$