

$$\begin{cases} f(n) \in [a, b] \\ f(n+r) \in [a, b] \end{cases} \Rightarrow \begin{cases} f(n) \in [a, b] & n > \omega \\ f(n+r) \in [a, b] & n \leq \omega \end{cases}$$

$$\Rightarrow f(n) - n + 1 > 0 \quad f(n) - n + 1 > 0 \cdot n > \omega \xRightarrow{(1)} n > 0 \cdot n > \omega \rightarrow n > \omega$$

$$f(n+r) - n + 1 > 0 \cdot n \leq \omega \xRightarrow{(2)} f(n+r) > 0 \rightarrow n > -\frac{r}{f}$$

(1) $n > \omega$

(2) $n > -\frac{r}{f} \text{ و } n \leq \omega \rightarrow -\frac{r}{f} \leq n \leq \omega \Rightarrow D_f = [-\frac{r}{f}, +\infty)$

$$\frac{f(n+r) - n + 1}{(f(n+r) - n + 1)^r} = \frac{f(n+r) - n + 1}{(f(n+r) - n + 1)^r} \Rightarrow \frac{f(n+r) - n + 1}{(f(n+r) - n + 1)^r}$$

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$D_f = \mathbb{R} - \{-\frac{1}{f}\}$

$$\begin{cases} n \neq \pm 1 & \frac{n+r-n-r}{n^r-1} = \frac{(n+r)(n+r)}{(n+r)(n+r)} \\ a = \pm 1 & \frac{a+r}{n+b} \end{cases}$$

$$\Rightarrow n=1 \rightarrow n+r=r \rightarrow \frac{a(1)+r}{(1)+b} = r \rightarrow \boxed{a = rb + 1}$$

$$n=-1 \rightarrow n+r=r-1 \rightarrow \frac{a(-1)+r}{(-1)+b} = 1 \rightarrow -a+r=b-1$$

$$a = rb + 1 \rightarrow -rb + 1 + r = b - 1 \rightarrow rb = r \rightarrow b = \frac{r}{r-1}$$

$$\Rightarrow g(n) = n+r \rightarrow c=r \quad b+c = \frac{1}{r-1} + r = \frac{\omega}{r} \quad a = \frac{r}{r-1} + 1 = \frac{\omega}{r}$$

۴ در صورت مسئله ۶ $a^2 - 1 \in \mathbb{Z}$ به $n+1$ تبدیل شده، پس به $n-1$

تقسیم شده یعنی $n=1$ ، پس صورت و مخرج $f(n)$ است. چون f تابع

برابرند پس دامنه های مخرج هم برابرند، پس $n=1$ ، پس مخرج $f(n)$

نیز هست، پس $n-c = n-1$ و $n^2 + an + b = (n-1)^2$

$$\Rightarrow n^2 - 2n + 1 = n^2 + an + b \rightarrow a = -2, b = 1, c = 1$$

$$a + b + c = 0$$

$$f(n+1) = (n+1-2)^2 (n+1-1) = (n-1)^2 (n)$$

$$f(-n+1) = (-n+1-2)^2 (-n+1-1)$$

$$n(n-1)^2 \cdot (-n)(-n+2)^2 = n(n^2 - (n+2) - n^2 - (n-2))$$

$$= n(-1n) = \boxed{-1n^2}$$

$$f(-1) \rightarrow 1 - \sqrt{n} = -1 \rightarrow \sqrt{n} = 2 \rightarrow n = 4 \Rightarrow f(-1) = 2 \quad (9)$$

$$f(3) \rightarrow 1 - \sqrt{n} = 3 \rightarrow \sqrt{n} = -2 \rightarrow n = 4 \quad f(3) = 1$$

$$f(-1) + f(3) = \underline{0}$$

$$n + \sqrt{n} = 4 \rightarrow b^2 + b - 4 = 0 \quad (b+3)(b-1) = 0 \quad b = 1 \quad (10)$$

$$\Rightarrow \sqrt{n} = 3 \rightarrow n = 9 \rightarrow f(4) = 1^2 + 1^2 = 2 \quad \boxed{2}$$

$$f(n) = n(n^2+1) \rightarrow f(n-1) = (n-1)((n-1)^2+1) \quad (11)$$

$$f(n-1) = (n-1)(n^2 - 2n + 2)$$

$$f(-n+1) = -(n-1)((-n+1)^2+1) = -(n-1)(n^2 - 2n + 2)$$

$$f(n-1) + f(-n+1) = 0$$

$$\text{Für } a=1 \quad f(n) \text{ und } g(n) \text{ sind } \sqrt{c-n} \text{ und } \sqrt{c-n} \quad (12)$$

$$D_g \Rightarrow g(n) = \sqrt{c-n} \rightarrow b = c$$

$$f(n) = \sqrt{c-n} \rightarrow ca = c \rightarrow a = \frac{c}{\mu}$$

$$f(x) = g \Rightarrow \cancel{K} \cdot \cancel{K} \rightarrow K = -1$$

$$K - 1 - K = K \rightarrow -K - 1 = K \rightarrow K = -1$$

$$\frac{1}{x} \cdot \frac{1}{x} = \frac{1}{x^2} + 1 = 2 \quad (13)$$

$$D_g = [-1, \infty)$$

$$D_f = (-\infty, 1] \Rightarrow m = 1$$

$$f(x) = x + n$$

$$g(x) = x + n - 1 \sqrt{x} = 4 \sqrt{x}$$

$$x + n = 4 \sqrt{x} \quad n = 4 \sqrt{x} - x \quad m + n = 4 + 1 \sqrt{x}$$