

A. $\frac{f(x)}{x^2}$ U. $\frac{f(x)}{x^2}$ K. $\frac{f(x)}{x^2}$

$$y = \sqrt{f(x) - x + 1}$$

$$f(x) = \begin{cases} x^2 - 1 & ; [2, \infty) \\ x^2 + 1 & ; (-\infty, 2) \end{cases}$$

①

$$f(x) - x + 1 \geq 0 \rightarrow f(x) \geq x - 1$$

$$\begin{cases} x \geq 2 \\ x < 2 \end{cases} \Rightarrow \begin{cases} x^2 - 1 \geq x - 1 \rightarrow x^2 \geq x \rightarrow x \geq 1 \text{ or } x \leq 0 \\ x^2 + 1 \geq x - 1 \rightarrow x^2 \geq x - 2 \rightarrow x \geq 1 \text{ or } x \leq -1 \end{cases}$$

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$$\textcircled{1} \cup \textcircled{2} \Rightarrow D_f = \left[-\frac{1}{x}, +\infty\right)$$

$$y = \frac{x^2 x^2 + x^2 x^2 + 1x + 1}{(x^2 + x + 1)^2} = \frac{x^2(1x^2 + 1x + 1)}{(x^2 + x + 1)^2} = \frac{x^2(x+1)^2}{(x+1)^2} \quad \textcircled{3}$$

$$\frac{1x^2 + 1x + 1}{x^2 + x + 1} \Bigg| \frac{x+1}{x^2 + x + 1}$$

$$x^2(x+1) = 4x+1$$

$$: x^2(x+1) = 4x+1$$

$$(x+1)^2 \neq 0 \rightarrow x+1 \neq 0$$

$$x \neq -1 \rightarrow x \neq -1$$

$$D_f = \mathbb{R} - \left\{-\frac{1}{x}\right\}$$

$$f(x) = \begin{cases} \frac{x^2 + x^2 - x - 1}{x^2 - 1} & ; x \neq \pm 1 \\ \frac{ax+r}{x+b} & ; x = \pm 1 \end{cases}$$

$$g(x) = x + c$$

$$b+c = ?$$

$$\frac{x^2 + x^2 - x - 1}{x^2 - 1} \xrightarrow{\text{long division}} \frac{(x-1)(x^2 + x + 1)}{x^2 - 1} \rightarrow \frac{(x-1)(x^2 + x + 1)}{(x+1)(x-1)} = \frac{(x^2 + x + 1)}{x+1}$$

$$f(1) = x \rightarrow x+1$$

$$f(-1) = 1 \rightarrow x-1$$

$$\frac{ax+r}{x+b} \xrightarrow{x=1} \frac{a+r}{1+b} = \frac{a+r}{1+b} = x+1$$

$$\xrightarrow{x=-1} 1 = \frac{-a+r}{-1+b}$$

$$-1+b = -a+r$$

$$r+fb = x$$

$$fb = x \rightarrow b = \frac{1}{x} \Rightarrow \frac{1}{x} = -a+r \rightarrow a, \frac{a}{x}$$

$$g(x) = x + c \rightarrow g(1) = x = 1 + c \rightarrow c = x$$

$$b+c = \frac{1}{x} + x = \frac{1+x^2}{x}$$

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$$f(x) = \frac{x+1}{x-c} \quad g(x) = \frac{x^2-1}{x^2+ax+b} \quad a+b+c=1 \quad (8)$$

$$\frac{x+1}{x-c} = \frac{(x+1)(x-1)}{x^2+ax+b} = \frac{x+1}{x-b} \rightarrow a-c = x-b \rightarrow b=c$$

$$(x-1)(x-b) = (x-1)^2 = x^2 - 2x + 1 \Rightarrow a = -2$$

$$g(x) = \frac{x^2-1}{(x-1)^2} = \frac{x+1}{x-1} \rightarrow x \neq 1 \Rightarrow c=1 \Rightarrow b=c=1$$

$$a+b+c = -2+1+1 = 0$$

$$f(x) = (x-1)^x (x-1) \quad g(x) = f(x+1) + f(1-x) \quad (9)$$

$$g(x) = (x-1)^x (x) + (-x-1)^x (-x) = (x^x - x^{x+1})x + (x^x + x^{x+1}) - x$$

$$= x(x^x - x^{x+1} - x^x - x^{x+1}) = x \cdot x - 1 \cdot x = -1 \cdot x^x$$

$$f(1-x) = \begin{cases} \sqrt{x+1} & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases} \quad f(-1) + f(1) \quad (10)$$

$$1-x = -1 \rightarrow x = 2 \rightarrow f(-1) = \sqrt{2} = 2$$

$$1-x = 1 \rightarrow x = 0 \rightarrow f(1) = \sqrt{1} = 1$$

$$f(-1) + f(1) = 3$$

$$f(x+\sqrt{x}) = x^x + x^x \quad f(4) = ? \quad (11)$$

$$x+\sqrt{x} = 4 \rightarrow x+\sqrt{x}-4=0 \rightarrow (\sqrt{x}-2)(\sqrt{x}+\frac{4}{\sqrt{x}}) = (\sqrt{x}-2)(\sqrt{x}+2) = 0$$

$$\begin{cases} \sqrt{x}-2 \rightarrow x=4 \\ \sqrt{x}+2 \rightarrow x \text{ is not real} \end{cases}$$

$$x=4 \rightarrow f(4) = 4^4 + 4^4 = 256 + 256 = 512$$

$$f(x) = x^x + x \quad f(x-1) + f(1-x) \quad (12)$$

$$= x(x^x+1) \rightarrow f(x-1) = (x-1)((x-1)^{x-1}+1)$$

$$f(1-x) = -(x-1)((1-x)^{1-x}+1)$$

$$f(x-1) + f(1-x) = (x-1)((x-1)^{x-1}+1) - (x-1)((1-x)^{1-x}+1) = 0$$

$$f(x) = \sqrt{x_1 + x_2} + x - 1$$

$$g(x) = \sqrt{b - x_2} + xk$$

(9)

$$xf - g = \{(1, k)\}$$

$$k - \frac{b}{x}k = ?$$

$$b - x_2 \geq 0 \rightarrow b \geq x_2 \rightarrow x_2 = 1$$

$$x=1 \quad x_1 = x_2 = a = \frac{x}{x}$$

$$b=2$$

$$xf - g \Rightarrow \sqrt{x_1 + x_2} + x - 1 - \sqrt{x_2} - xk = -k - x = k$$

$$-x = 2k \rightarrow k = -1$$

$$x_2 = \frac{b}{x} - k = x \times \frac{x}{x} - \frac{x}{x} + 1 = x - x + 1 = 1$$

$$f(x) = \sqrt{m - x} + x$$

$$g(x) = \sqrt{x_2 + x}$$

(10)

$$D_{f \times g} = [1, \sqrt{v}]$$

$$f \cdot g(x) = 4\sqrt{x}$$

$$D_f = m - x \geq 0$$

$$D_g = x_2 + x \geq 0$$

$$m \geq x$$

$$x_2 + x \geq -1$$

$$\Rightarrow m = v$$

$$f - g(x) = 4\sqrt{x} = \sqrt{v - x} + x - \sqrt{x_2 + x} = x + n - \sqrt{x} = x + n - x\sqrt{x} = 4\sqrt{x}$$

$$x + n = 4\sqrt{x}$$

$$n = 4\sqrt{x} - x$$

$$m + n = v + 4\sqrt{x} - x = 0 + 4\sqrt{x}$$

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