

سوال ۱

$$f(x) = \sqrt{f(x) - x + 1}$$

$$f(x) = \begin{cases} 2x-1 & [x] > 2 \rightarrow a < x \\ x+3 & [x] \leq 2 \rightarrow a > x \end{cases}$$

$$\Rightarrow \sqrt{a-2+1} = \sqrt{a}$$

$$\sqrt{2x+1} = \sqrt{x} \rightarrow 2x+1 \geq 0 \rightarrow x \geq -\frac{1}{2} \rightarrow D_f: [-\frac{1}{2}, +\infty)$$

$$f(x) = \frac{2x^3 + 3x^2 + 1}{(x^2 + x + 1)^2} \rightarrow \frac{3(2x+1)^2}{(2x+1)^2} \xrightarrow{2x+1 \neq 0 \rightarrow x \neq -\frac{1}{2}} 3(2x+1)$$

سوال ۲

$$\Rightarrow D \cap R = \{-\frac{1}{2}\}$$

$$f(x) = \begin{cases} \frac{x^2 + 2x^2 - x - 2}{x^2 - 1} & , x \neq \pm 1 \\ \frac{ax+b}{x+b} & , x = \pm 1 \end{cases}$$

$$g(x) = x + c \quad b + c = ?$$

سوال ۳

$$\begin{aligned} \textcircled{1} \frac{-a+2}{-1+b} &= 1 \rightarrow 2-a-b-1 \rightarrow a=1-b \\ \textcircled{2} \frac{a+2}{1+b} &= 3 \rightarrow a+2=3b+3 \rightarrow a=3b+1 \end{aligned} \Rightarrow$$

$$\frac{x^2 + 2x^2 - x - 2}{x^2 - 1} = \frac{(x^2 - 1)(x+2)}{x^2 - 1} = x+2$$

$$1+3b=3-b \rightarrow b=\frac{2}{4}=\frac{1}{2}$$

$$\Rightarrow b+c=\frac{3}{2}$$

$$f(x) = \frac{x+1}{x-c}, \quad g(x) = \frac{x^2-1}{x^2+ax+b}$$

$$a+b+c=? \Rightarrow -(-1)+c+c=c-1 \stackrel{c=1}{=} 0$$

سوال ۴

$$\frac{x+1}{x-c} = \frac{x^2-1}{x^2+ax+b} \rightarrow (x-c)(x-1)(x+1) = (x+1)(x^2+ax+b) \rightarrow (x-c)(x-1) = x^2+ax+b$$

$$\Rightarrow x^2 + (-c-1)x + c = x^2 + ax + b \Rightarrow \begin{cases} a = -c-1 \\ b = c \end{cases}$$

☆ تابع دایره ای (مربعی) مساوی اند به دو دایره مساوی باشند
 $n-c=n-1$ و n^2+an+b
 این معادله دارد

$$f(x) = (x-2)^2(x-1) \quad g(x) = f(x+1) + f(1-x)$$

سوال ۵

$$g(x) = f(x+1) + f(1-x) \rightarrow x(x-2)^2 + (x+2)^2(-x) \rightarrow x(x^2-4x+4) - x(x^2+4x+4) = -8x^2$$

$$f(1-2x) = \begin{cases} \sqrt[3]{x} + 2 & x \leq 0 \\ x\sqrt{x} & x > 0 \end{cases} \quad f(-1) + f(3) = ?$$

سوال ۶

$$\begin{aligned} f(-1) &\Rightarrow 1-2x=-1 \rightarrow 2x=2 \rightarrow x=1 \xrightarrow{1>0} \sqrt[3]{1} = 1 \\ f(3) &\Rightarrow 1-2x=3 \rightarrow x=-1 \xrightarrow{-1<0} \sqrt[3]{-1} + 2 = 1 \end{aligned} \rightarrow \textcircled{0}$$

$$f(x+\sqrt{x}) = x^2 + 3x \quad f(4) = ?$$

سوال ۷

$$x + \sqrt{x} = 9 \rightarrow x=4 \quad x^2 + 3x \xrightarrow{x=4} 4^2 + 3(4) = 16 + 12 = 28$$

$$f(x) = x^3 + x \quad f(x-1) + f(1-x) = ?$$

سوال ۸

$$\Rightarrow (x-1)(x^2-2x+2) + (1-x)(x^2-2x+2) = 0$$

$$x^3 + x = x(x^2+1) \quad (x-1)^2 + 1 = x^2 - 2x + 2 \quad (1-x)^2 + 1 = x^2 - 2x + 2$$

سوال ۹

$$f(x) = \sqrt{x-3}a + K - 1, \quad g(x) = \sqrt{b-4}x + 3K \quad 2f-g = \{(1, K)\}$$

$$3a - \frac{b}{3} - K = ?$$

در ادامه فقط ۱ مشترک بوده است $\Leftarrow$ راجعاًل هر دو در $x=1$ برابر صفر می شود $\Leftarrow$

$$\begin{cases} b - f(1) = 0 \rightarrow b - 4 = 0 \rightarrow b = 4 \\ f(1) - 3a = 0 \rightarrow 1 - 3a = 0 \rightarrow 1 = 3a \rightarrow a = \frac{1}{3} \end{cases}$$

$$2f-g \rightarrow 2\sqrt{1-3}a + 2K - 1 - \sqrt{b-4} - 3K = K \rightarrow -K - 1 = K \rightarrow 2K = -1 \rightarrow K = -\frac{1}{2}$$

$$3a - \frac{b}{3} - K = 3\left(\frac{1}{3}\right) - \frac{4}{3} - (-\frac{1}{2}) = 1 - \frac{4}{3} + \frac{1}{2} = \frac{2}{3} - \frac{4}{3} + \frac{1}{2} = -\frac{2}{3} + \frac{1}{2} = -\frac{4}{6} + \frac{3}{6} = -\frac{1}{6}$$

سوال ۱۰

$$f(x) = \sqrt{m-x} + n, \quad g(x) = \sqrt{2x+2}, \quad D_{f \times g} = [-1, 7], \quad f-g(4) = 4\sqrt{2}$$

$$m+n = ?$$

$$f(x) = \sqrt{m-x} + n \rightarrow m-x \geq 0 \rightarrow x \leq m \Rightarrow m=7$$

$$g(x) = \sqrt{2x+2} \rightarrow 2x+2 \geq 0 \rightarrow 2x \geq -2 \rightarrow x \geq -1$$

$$D_f \cap D_g = [-1, 7], \quad f-g(4) = 2+n - 2\sqrt{2} = 4\sqrt{2} \rightarrow 2+n = 6\sqrt{2} \rightarrow n = 6\sqrt{2} - 2$$

$$m+n = 7 + 6\sqrt{2} - 2 = 5 + 6\sqrt{2}$$