

1- $y = \sqrt{f(n) - n + 1}$

$f(n) = \begin{cases} n-1 & ; [n] > \frac{1}{2} \rightarrow n \geq 2 \\ n+1 & ; [n] \leq \frac{1}{2} \rightarrow n \leq 1 \end{cases}$

$\sqrt{f(n-1)-n+1} = \sqrt{n-1-n+1} = \sqrt{-1} < 0 \rightarrow n \geq 2$

$\sqrt{f(n+1)-n+1} = \sqrt{n+1-n+1} = \sqrt{1} = 1 > 0 \rightarrow n \leq 1$

$\therefore D_f = [-\frac{1}{2}, +\infty)$

2- $y = \frac{r^n + c^n + 1}{(r^n + c^n + 1)^2} = \frac{r(1 + 1 + 1)}{(r^n + c^n + 1)^2} = \frac{r(1)^n}{(r^n + c^n + 1)^2} = \frac{r}{r^n + 1}$

$\rightarrow D_f = \mathbb{R} - \{-\frac{1}{r}\}$

3- $f(n) = \begin{cases} \frac{n^2 + n - 1}{n^2 - 1} & ; n \neq 1 \\ \frac{a + r}{n + b} & ; n = 1 \end{cases}$

$\rightarrow \frac{x^2(n+r)-(n+1)}{n^2-1} = \frac{(x^2-1)(n+r)}{n^2-1} \rightarrow g(n) = n + c$

$\begin{cases} r = 1 + c, 1 = 1 + c \\ n + r = n + c + c = 2c \end{cases} \rightarrow c = \frac{1}{2}$

$\frac{a+r}{1+b} = 1 \rightarrow a+r = 1+b \rightarrow a-b = 1-b-r$

$\frac{-a+r}{-1+b} = 1 \rightarrow -a+r = -1+b \rightarrow a-b = r-1$

$\Rightarrow a-b = 1-b-r \Rightarrow a = 1-b-r$

$\Rightarrow a = 1-b-r$

4- $g(n) = \frac{n^2-1}{n^2+an+b}$ $f(n) = \frac{n+1}{n-c}$

$D_f = \mathbb{R} - \{c\} \Rightarrow D_g = \mathbb{R} - \{c\} \rightarrow D \neq \emptyset \Rightarrow a^2 - 4b \geq 0$

$\frac{(n-1)(n+1)}{(n^2+an+b)} = \frac{n+1}{n-c} \rightarrow n^2+an+b = (n-1)(n-c) \Rightarrow a = -2, b = 1$

$c = \frac{1}{2}$

$\Rightarrow a+b+c = -2+1+\frac{1}{2} = -\frac{1}{2}$

5- $f(n) = (n-1)^n (n-1)$

$f(n+1) + f(1-n) = (n-1)^n (1-n) + (-n-1)^n (-n) = ((n-1)^n - (n+1)^n) = n(-1)(n) = -n^2$

$\Rightarrow g(n) = -n^2$

6- $f(1-n) = \begin{cases} \sqrt{n} + 1 & n \leq 0 \\ \sqrt{n} & n > 0 \end{cases}$

$n=1 \rightarrow f(-1) = 1$

$n=-1 \rightarrow f(1) = 1$

$\Rightarrow f(-1) + f(1) = 2$

7. $f(x+\sqrt{x}) = x^{\frac{1}{2}} + \sqrt{x}$

$$x+\sqrt{x} \geq 4 \rightarrow x+\sqrt{x}-4 \geq 0$$

$$\Rightarrow (\sqrt{x}-1)(\sqrt{x}+4) = 0 \rightarrow \begin{cases} \sqrt{x} \geq 1 \\ \sqrt{x} \geq 4 \Rightarrow x \geq 16 \end{cases}$$

$$\Rightarrow f(4) = 2^{\frac{1}{2}} + 2 = 2\sqrt{2} + 2$$

8. $f(n) = n^{\frac{1}{2}} + n = n^{\frac{1}{2}}(n^{\frac{1}{2}} + 1)$

$$f(n-1) + f(1-n) = (n-1)(n^{\frac{1}{2}} - 1 + 1) + (1-n)(n^{\frac{1}{2}} - 1 + 1) = (n-1)(n^{\frac{1}{2}} - 1 + 1)(1-1) = 0$$

$$\Rightarrow f(n-1) + f(1-n) = 0$$

9. $f(n) = \sqrt{a-n} + k - 1$

$$g(n) = \sqrt{b-fn} + k$$

$$f \circ g = \{(1, k)\}$$

$$\rightarrow f \circ g(1) = k$$

$$\rightarrow f \circ g(1) = \sqrt{a-f} + k - 1 = \sqrt{b-f} + k \Rightarrow k = k$$

$$\sqrt{a-f} - \sqrt{b-f} - 1 = 0$$

$$\Rightarrow a = \frac{b}{f}, b = f, k = -1 \Rightarrow a - \frac{b}{f} - k = f - 1 + 1 = f$$

10. $f(n) = \sqrt{m-n} + n$

$$g(n) = \sqrt{n+1}$$

$$D_{f \circ g} = [-1, \sqrt{V}]$$

$$f \circ g(\sqrt{V}) = 4\sqrt{V}$$

$$D_g = [-1, +\infty)$$

$$D_f = (-\infty, m]$$

$$\xrightarrow{\cap} [-1, \sqrt{V}] \Rightarrow m = V$$

$$\Rightarrow f \circ g(\sqrt{V}) = \sqrt{V-1} + n - \sqrt{1} = 4\sqrt{V} \Rightarrow 4n - \sqrt{V} = 4\sqrt{V} \Rightarrow n = \sqrt{V} + 1$$

$$\Rightarrow m + n = V + \sqrt{V} + 1 = \sqrt{V} + 1$$

(۲) ضابطه های بیساز

$$f(n) = \begin{cases} \frac{(n-1)(n^2+3n+2)}{(n-1)(n+1)} = n+2 \\ \frac{an+2}{n+b} \end{cases}$$

$$f(1) = \frac{a+2}{1+b} = 3 \rightarrow a-3b = 1$$

$$f(-1) = \frac{-a+2}{-1+b} = 1 \rightarrow a+b = 3$$

۳- برای تساوی دو تابع (۱) دامنه های برابر

$$n \neq \mp 1 \Rightarrow c = 2$$

$$n = \mp 1$$

مقدار تابع در $n = \mp 1$ باید در دو تابع f و g برابر باشند:

$$\left\{ \rightarrow a = 2, 5, b = 0, 5 \rightarrow \boxed{b+c = 2, 5} \right.$$