

1- $y = \sqrt{f(n) - n + 1}$

$$f(n) = \begin{cases} n-1 & ; [n] > \frac{1}{2} \rightarrow n \geq 2 \\ n+1 & ; [n] < \frac{1}{2} \rightarrow n < 2 \end{cases}$$

$$\sqrt{n-1-n+1} = \sqrt{-1} \rightarrow \text{not real} \quad \text{for } n \geq 2$$

$$\sqrt{n+1-n+1} = \sqrt{1} \rightarrow \text{real} \quad \text{for } n < 2$$

$\therefore D_f = [-\frac{1}{2}, +\infty)$

2- $y = \frac{r^n + c r^{n-1} + k n + r}{(r^n + r n + 1)^r} = \frac{r(1 + \frac{c}{r} + \frac{k}{r} n + 1)}{(r^n + r n + 1)^r} = \frac{r(r n + 1)^r}{(r n + 1)^r} = \frac{r}{r n + 1}$

$\rightarrow D_f = \mathbb{R} - \{-\frac{1}{r}\}$

3- $f(n) = \begin{cases} \frac{n^r + r n^{r-1} - n - r}{n^r - 1} & ; n \neq 1 \\ \frac{a n + r}{n + b} & ; n = 1 \end{cases}$

$\rightarrow \frac{x^r(n+r) - (n+r)}{n^r - 1} = \frac{x^r - 1}{n^r - 1} g(n) = n + c$

$\begin{cases} r = 1 + c, & 1 = -1 + c \\ n + r = n + c + c = 2c \end{cases} \Rightarrow$

$\frac{a+r}{1+b} = r \rightarrow a+r = r+b \rightarrow a-b = 1$

$\frac{-a+r}{-1+b} = 1 \rightarrow -a+r = -1+b \rightarrow a+b = r$

$\Rightarrow \begin{cases} a = r, & b = 1 \\ b + c = r \end{cases}$

4- $g(n) = \frac{n^r - 1}{n^r + a n + b} \quad f(n) = \frac{n+1}{n-c}$

$\exists D_f \subset \mathbb{R} - \{c\} \Rightarrow D_g \subset \mathbb{R} - \{c\} \rightarrow \Delta \neq 0 \Rightarrow a^r - r b = 0$

$\frac{(n-1)(n+1)}{(n^r + a n + b)} = \frac{n+1}{n-c} \rightarrow n^r + a n + b = (n-1)(n-c) \quad \Rightarrow a = -r, b = 1$

$(n^r - r n + b) \Rightarrow c = 1$

$\Rightarrow a + b + c = -r + 1 + 1 = 0$

5- $f(n) = (n-r)^r (n-1)$

$f(n+1) + f(1-n) = (n-r)^r (1+n) + (-n-r)^r (-n) = ((n-r)^r - (n+r)^r) = n(-r)(r) = -r n^2$

$\Rightarrow g(n) = -r n^2$

6- $f(1-n) = \begin{cases} \sqrt{n} + r & n \leq 0 \\ r \sqrt{n} & n > 0 \end{cases}$

$n=1 \rightarrow f(-1) = r \quad \Rightarrow f(-1) + f(1) = 2r$

$n=-1 \rightarrow f(r) = 1$

7. $f(x+\sqrt{x}) = x^{\frac{1}{2}} + \sqrt{x}$

$$x+\sqrt{x} \geq 4 \rightarrow x+\sqrt{x}-4 \geq 0$$

$$\Rightarrow (\sqrt{x}-1)(\sqrt{x}+1) = 0 \rightarrow \begin{cases} \sqrt{x} \geq 1 \\ \sqrt{x} \geq 1 \Rightarrow x \geq 1 \end{cases}$$

$$\Rightarrow f(1) = 1^{\frac{1}{2}} + 1 = 2$$

8. $f(n) = n^{\frac{1}{2}} + n = n^{\frac{1}{2}}(n^{\frac{1}{2}} + 1)$

$$f(n-1) + f(1-n) = (n-1)^{\frac{1}{2}}(n^{\frac{1}{2}} - 1 + 1) + (1-n)^{\frac{1}{2}}(n^{\frac{1}{2}} - 1 + 1) = (n-1)^{\frac{1}{2}}(n^{\frac{1}{2}} - 1 + 1)(1-1) = 0$$

$$\Rightarrow f(n-1) + f(1-n) = 0$$

9. $f(n) = \sqrt{a-n} + k - 1$

$$g(n) = \sqrt{b-n} + k$$

$$f-g = \{(1, k)\}$$

$$\rightarrow f-g(1) = k$$

$$\rightarrow f-g(1) = \sqrt{a-1} + k - 1 - \sqrt{b-1} + k = k$$

$$\sqrt{a-1} - \sqrt{b-1} - 1 + k = k$$

$$\Rightarrow \sqrt{a-1} - \sqrt{b-1} = 1 \Rightarrow a - \frac{b}{4} - k = 1 + 1 = 2$$

10. $f(n) = \sqrt{m-n} + n$

$$g(n) = \sqrt{n+1}$$

$$D_{f \times g} = [-1, V]$$

$$f-g(n) = 4\sqrt{2}$$

$$D_g = [-1, +\infty)$$

$$D_f = (-\infty, m]$$

$$\rightarrow [-1, V] \Rightarrow m = V$$

$$\Rightarrow f-g(V) = \sqrt{V-1} + n - \sqrt{1} = 4\sqrt{2} \Rightarrow 1+n - \sqrt{1} = 4\sqrt{2} \Rightarrow n = 4\sqrt{2} - 1$$

$$\Rightarrow m+n = V + 4\sqrt{2} - 1 = 4\sqrt{2} + 1$$