

$$1) D_f y=8 \rightarrow y = \sqrt{f(x) - x + 1}$$

$$\text{if } f(x) = \begin{cases} x-1 & ; [x] > 8 \\ x+8 & ; [x] \leq 8 \end{cases}$$

$$f(x) - x + 1 \geq 0$$

$$[x] > 8 \rightarrow x \geq 9$$

$$\begin{aligned} x-1-x+1 &\geq 0 \rightarrow D_f = [9, +\infty) \\ x &\geq 0 \cap x \geq 9 \end{aligned}$$

$$[x] \leq 8 \rightarrow x < 9$$

$$x+8-x+1 \geq 0$$

$$x+8 \geq 0 \rightarrow x \geq -\frac{8}{1}, x < 9 \rightarrow D_f = [-\frac{8}{1}, 9)$$

$$\sim I \cup II \rightarrow [-\frac{8}{1}, 9) \cup [9, +\infty) \rightarrow [-\frac{8}{1}, +\infty)$$

$$r) y = \frac{r \varepsilon n^r + r^2 n^r + 1 n n + r}{(\varepsilon n^r + \varepsilon n + 1)^r} \rightarrow \frac{r \varepsilon n^r + r^2 n^r + 1 n n + r}{(r n + 1)^r}$$

$$\Rightarrow r \varepsilon n^r + r^2 n^r + 1 n n + r$$

$$\Rightarrow r (1 n^r + r n^r + n + 1)$$

$$\frac{r (r n + 1)^r}{(r n + 1)^r} \rightarrow \frac{r}{r n + 1} \Rightarrow r n + 1 \neq 0 \rightarrow r n \neq -1$$

$$x \neq -\frac{1}{r}$$

$$D_f = \mathbb{R} \setminus \left\{ -\frac{1}{r} \right\}$$

$$r) f(n) = \begin{cases} \frac{n^r + r n^r - n - r}{n^r - 1} ; n \neq \pm 1 \\ \frac{a n + r}{n + b} ; n = \pm 1 \end{cases}$$

$$g(n) = n + c \quad b + c = r$$

$$\rightarrow (x^r + r x^r) + (-x - r) \rightarrow x^r (n + r) - (x + r) \rightarrow (x + r) (n^r - 1)$$

$$\frac{x^r + r x^r - x - r}{x^r - 1} = \frac{(x + r) (n^r - 1)}{n^r - 1} = (x + r) \cdot 0$$

$$n + r = n + c \rightarrow (c = r)$$

$$x = 1 \rightarrow \frac{a + r}{1 + b} \rightarrow \frac{1}{1 + b} = r \rightarrow \frac{a + r}{1 + b} = r \rightarrow r b + r = a + r$$

$$a = r b + 1$$

$$x = -1 = \frac{-a + r}{b - 1} \Rightarrow -1 + r = +1 \Rightarrow \frac{-a + r}{b - 1} = +1 \rightarrow -a + r = b - 1$$

$$a = r - b$$

$$r b + 1 = r - b \rightarrow (b = \frac{1}{r})$$

$$b^2 + c^2 = r^2$$

$$\Leftarrow) f(x) = \frac{x+1}{x-c}$$

$$g(x) = \frac{x^r - 1}{x^r + ax + b} \rightsquigarrow (x+1)(x-1)$$

$$(x-c)(x-1) = x^r + \underset{\downarrow}{a}x + \underset{\downarrow}{b}$$

$$\begin{matrix} (x-1)(x-c) \\ \downarrow D_f \{-1\} \end{matrix}$$

$$a = c+1, b = c$$

$$x < \rightarrow D_f = \mathbb{R} - \{1\} \rightarrow c = \underline{1}$$

$$\left. \begin{matrix} c=1 \\ b=1 \\ a=-2 \end{matrix} \right\} \xrightarrow{+} \underline{0}$$

$$\omega) f(x) = (x+r)^r(x-1)$$

$$g(x) = f(x+1) + f(1-x)$$

$$f(x+1) \rightarrow (x+1-\frac{r}{r})^r(x+1-1) \rightarrow (x-r)^r(x) \rightarrow (x^r - \varepsilon x + \varepsilon)x$$

$$f(1-x) \rightarrow (1-x-r)^r(1-x-1)$$

$$\downarrow$$

$$(-x-r)^r(-x) \rightarrow (x^r + \varepsilon x + \varepsilon)(-x)$$

$$\begin{matrix} r \\ \leftarrow 2x + \varepsilon \end{matrix}$$

$$\boxed{-x^r - \varepsilon x^r - \varepsilon x}$$

$$\cancel{x^r - \varepsilon x^r + \varepsilon x} - \cancel{x^r - \varepsilon x^r - \varepsilon x} = \underline{\underline{-2x^r}}$$

$$4) f(-1) + f(x) \rightsquigarrow f(1-x) = \begin{cases} \sqrt[3]{x} + 1 & x \leq 0 \\ \varepsilon \sqrt{x} & x > 0 \end{cases}$$

$$f(-1) \rightsquigarrow f(1-x) = f(-1)$$

$$-1 = 1-x \rightsquigarrow 1-x+1=0 \rightarrow x(1-x)=0 \\ x-x^2=0 \quad x=0$$

$$x \rightarrow 1 \rightarrow \varepsilon \sqrt{1} \rightarrow \underline{\underline{x}} \rightsquigarrow f(-1) = \underline{\underline{x}}$$

$$f(x) \rightsquigarrow f(1-x) = f(x)$$

$$x = 1-x \rightsquigarrow 1-x-x=0 \rightsquigarrow -x-x=0$$

$$x \rightarrow -1 \rightsquigarrow \sqrt[3]{-1} + 1 = \underline{\underline{0}} \rightsquigarrow f(x) = \underline{\underline{1}} \quad -x(1+x)=0 \\ x = \underline{\underline{-1}}$$

$$f(-1) + f(x) = x + 1 = \underline{\underline{0}}$$

$$V) \underbrace{f(4)}_2 \rightsquigarrow f(x+\sqrt{x}) = x^x + x^x$$

$$\underbrace{(x)^x}_2 + \underbrace{x(x)}_{12} \Rightarrow \boxed{14}$$

$$x + \sqrt{x} = 4 \rightarrow \boxed{x=4} \rightsquigarrow (\sqrt{x})^x + \sqrt{x} - 4 = 0$$

$$\begin{aligned} & \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{x} \\ & \downarrow \\ & \sqrt{x}=2 \quad \sqrt{x}=-3 \text{ (reject)} \\ & \boxed{x=4} \end{aligned}$$

$$A) f(x) = x^x + x$$

$$f(x-1) + f(1-x) \rightsquigarrow f(x-1) = (x-1)^{x-1} + x-1$$

$$f(1-x) = (1-x)^{1-x} + 1-x$$

$$\Rightarrow \left(\frac{x-1}{0}\right)^{x-1} + x-1 + (1-x)^{1-x} + 1-x \neq \underline{\underline{0}}$$

منظور سوال را متوجه نمیشم حاصلش برای
 باشد مصفا شده بازه اس؟ چه؟
 روی نمودار

$$9) f(x) = \sqrt{ax - 3a} + k - 1 \quad 2f - g \quad \text{نویسه}$$

$$g(x) = \sqrt{b - cx} + 3k$$

$$2f - g = \{(1, k)\}$$

$$3a - \frac{b}{c} - k = 0$$

$$2f - g \rightsquigarrow \left. \begin{aligned} 2f(x) &= 2\sqrt{ax - 3a} + 2k - 2 \\ g(x) &= \sqrt{b - cx} + 3k \end{aligned} \right\} \begin{aligned} 2\sqrt{ax - 3a} - \sqrt{b - cx} - k - 2 \end{aligned}$$

$$10) f(x) = \sqrt{m - x} + n$$

$$g(x) = \sqrt{2x + 2}$$

$$D_{f \times g} = [-1, 4] \rightsquigarrow$$

$$f - g(x) = 4\sqrt{x}$$

$$m + n = 2$$

$$2x + 2 \geq 0 \rightarrow x \geq -1 \rightsquigarrow g(x) \leq M$$

$$\sqrt{2x + 2} \leq M$$

$$\rightarrow \sqrt{2(4)} + 2 \leq M$$

$$(m = 4)$$

$$f - g(x) = 4\sqrt{x} \rightarrow f(x) - g(x) = 4\sqrt{x}$$

$$g(x) = \sqrt{2(x) + 2} = 2\sqrt{x}$$

$$f(x) = \sqrt{m - x} + n \xrightarrow{m=4} \sqrt{4 - x} + n = 1 + n$$

$$1 + n - 2\sqrt{x} = 4\sqrt{x} \rightarrow n = 8\sqrt{x} - 1 \rightsquigarrow$$

$$m + n = 4 + 8\sqrt{x} - 1 \rightarrow 3 + 8\sqrt{x}$$