

دواریم فرض

کلیف K

نیاسیه ساه تقری

$$y = \sqrt{f(n) - n + 1}$$

$$f(n) = \begin{cases} Kn-1 & [n] > K \rightarrow n \geq \Delta \\ K_{n+K} & [n] \leq K \rightarrow n < \Delta \end{cases} \quad (1)$$

$$f(n) - n + 1 \geq 0 \rightarrow f(n) \geq n - 1 \rightarrow Kn - 1 \geq n - 1 \rightarrow n \geq 0 \quad \checkmark$$

(1) $n \geq \Delta$

$n < \Delta \rightarrow K_{n+K} \geq n - 1 \rightarrow Kn \geq -K \rightarrow n \geq -\frac{K}{K}$

$$(1) \cap (2) \rightarrow D_f = \left[-\frac{K}{K}, +\infty\right)$$

(2) $-\frac{K}{K} \leq n < \Delta$

$$y = \frac{r^n + r^{n+1} + 1}{(r^n + r^{n+1})^r} = \frac{r(1 + r + 1)}{(r^{n+1})^r} \quad (1)$$

$$\frac{r(r^{n+1})^r}{(r^{n+1})^r} = \boxed{\frac{r}{r^{n+1}}} \quad (r^n + r^{n+1})^r \neq 0$$

$$(r^{n+1})^r \neq 0$$

$$r^{n+1} \neq 0 \rightarrow n \neq -\frac{1}{r}$$

$$D_f = \mathbb{R} - \left\{-\frac{1}{r}\right\}$$

$$f(n) = \begin{cases} \frac{n^r + r^n - n - r}{n^r - 1} & n \neq \pm 1 \\ \frac{a + r}{n + b} & n = \pm 1 \end{cases} \quad (2)$$

$$g(n) = n + c = n + r$$

$$\frac{n^r + r^n - n - r}{n^r - 1} = \frac{(n^r - n) + r(n^r - 1)}{n^r - 1} = \frac{(n + r)(n^r - 1)}{(n^r - 1)} = n + r \rightarrow \boxed{c = r}$$

$$n = 1 \rightarrow g(n) = f(n) = r = \frac{a + r}{b + 1} \rightarrow r(b + 1) = a + r$$

$$\boxed{r \cdot b + 1 = a}$$

$$n = -1 \rightarrow g(n) = f(n) = 1 = \frac{r - a}{b - 1} \rightarrow r - a = b - 1$$

$$\boxed{r - b = a}$$

$$r - b = r \cdot b + 1 \rightarrow \boxed{b = \frac{1}{r}} \quad a = r \cdot b + 1 = \boxed{\frac{a}{r}}$$

$$b + c = r + \frac{1}{r} = \boxed{\frac{a}{r}} \quad \checkmark$$

$$f(n) = \frac{n + 1}{n - c} \quad g(n) = \frac{n^r - 1}{n^r + a n + b} \quad (3)$$

نکته: c و b و a و r در $g(n)$ باید همبستگی داشته باشند.



$$\frac{n+1}{n-c} \times \frac{(n-1)}{(n-1)} = \frac{n^r - 1}{(n-c)(n-1)} = \frac{n^r - 1}{n^r + an + b} \Rightarrow$$

$$n-c = n-1 \rightarrow \boxed{c=1} \rightarrow (n-1)^r = n^r - r n^{r-1} + \dots = n^r + an + b \Rightarrow$$

$$\boxed{a=-r} \quad \boxed{b=1} \rightarrow \boxed{a+b+c=0}$$

$$f(n) = (n-r)^r (n-1) \quad \textcircled{2}$$

$$g(n) = f(n+1) + f(1-n)$$

$$\rightarrow f(n+1) = ((n+1)-r)^r ((n+1)-1) \Rightarrow f(n+1) = (n-r)^r (n)$$

$$\rightarrow f(1-n) = ((1-n)-r)^r ((1-n)-1) \Rightarrow f(1-n) = (n+r)^r (-n)$$

$$g(n) = (n-r)^r (n) - (n+r)^r (n) = n^r - r n^{r-1} + r n - n^r - r n^{r-1} - r n = \boxed{-2 r n^{r-1}}$$

$$f(1-rn) = \begin{cases} \sqrt[r]{rn} + r & n \leq 0 \\ r\sqrt[r]{n} & n > 0 \end{cases} \quad \textcircled{3}$$

$$f(-1) \rightarrow 1-rn = -1 \rightarrow n=1 \rightarrow \sqrt[r]{1} = 1$$

$$f(r) \rightarrow 1-rn = r \rightarrow n=-1 \rightarrow \sqrt[r]{-1} + r = 1$$

$$f(-1) + f(r) = 1$$

$$f(n+\sqrt{n}) = n^r + r\sqrt{n} \quad \textcircled{4}$$

$$f(4) \rightarrow n+\sqrt{n}=4 \rightarrow \sqrt{n}=t \rightarrow t^2+t-4=0 \rightarrow t=2 \rightarrow \sqrt{n}=2 \rightarrow n=4$$

$$\rightarrow t=-3 \quad \text{قبح}$$

$$n^r + r\sqrt{n} = 2 \cdot 4 + 1 \cdot 2 = \boxed{10}$$

$$f(n) = n^r + n$$

(A)

$$f(n-1) + f(1-n) \Rightarrow f(n-1) = (n-1)^r + (n-1)$$

$$f(1-n) = (1-n)^r + (1-n)$$

$$\Rightarrow f(1-n) = -(n-1)^r - (n-1)$$

$$f(n-1) + f(1-n) = \cancel{(n-1)^r} + (n-1) - \cancel{(n-1)^r} - (n-1) = 0$$

$$f(x) = \sqrt{kx - ka} + k - 1 \rightarrow kx - ka \geq 0 \rightarrow x \geq \frac{k}{k} a$$

(9)

$$g(x) = \sqrt{b - kx} + k \rightarrow b - kx \geq 0 \rightarrow x \leq \frac{b}{k}$$

$$D_{f \cap g} = \{1\} \rightarrow \frac{k}{k} a = \frac{b}{k} = 1 \Rightarrow \boxed{b = k} \quad \boxed{a = \frac{k}{k}}$$

$$f(1) - g(1) = 2 \left(\sqrt{k - k \left(\frac{k}{k} \right)} + k - 1 \right) - \left(\sqrt{k - k} + k \right) = k$$

$$2k - 2 - k = k \rightarrow -2 - k = k \rightarrow 2k = -2$$

$$\boxed{k = -1}$$

$$ka - \frac{b}{k} - k = 2 \left(\frac{k}{k} \right) - \frac{k}{k} - (-1) = 2$$

$$f(x) = \sqrt{m - x} + x \rightarrow D_f = m - x \geq 0 \rightarrow x \leq m$$

(10)

$$g(x) = \sqrt{kx + r} \rightarrow D_g = kx + r \geq 0 \rightarrow x \geq -1$$

$$D_{f \cap g} = D_f \cap D_g = [-1, v] \rightarrow -1 \leq x \leq v \rightarrow \boxed{m = v}$$

$$f(r) - g(r) = 4\sqrt{r} \rightarrow \sqrt{v - r} + r - \sqrt{k} = 4\sqrt{r} \rightarrow r + n = 4\sqrt{r}$$

$$\boxed{n = 4\sqrt{r} - r}$$

$$\boxed{m + n = 4\sqrt{r} + 2}$$