

$\mu: \begin{cases} 2n-1 & n \geq 0 \\ 2n+3 & n < 0 \end{cases}$
 $n \geq 0 \rightarrow f(n) - n + 1 \geq 0 \Rightarrow 2n-1 - n + 1 \geq 0 \Rightarrow n \geq 0 \cap (n \geq 0) = n \geq 0$ (1)
 $n < 0 \rightarrow 2n+3 - n + 1 \geq 0 \Rightarrow n \geq -2, n < 0 = [-2, 0)$ (2)

$(1) \cup (2) = [-2, +\infty) = D_f$

$$\frac{2 \sum n^2 + 3 \sum n + 1 \sum 1}{(n+1)^3} = \frac{2(n+1)(\sum n^2 - \sum_{k=1}^n k^2) + 1(n+1)(\sum_{k=1}^n k)}{(n+1)^3} \Rightarrow$$

$$\frac{(n+1)(2n^2 + 3n + 1)}{(n+1)^3} = \frac{2n^2 + 3n + 1}{(n+1)^2}$$

$D_f = \mathbb{R} - \{-\frac{1}{2}\}$

(استفاده از f را برای $n \neq \pm 1$ می‌کنیم)
 $\frac{(n-1)(n+2)(n+1)}{(n-1)(n+1)} = n+2$
 $g(1) = 3 = \frac{a+2}{1+b} \rightarrow 3b-a = -1$
 $g(-1) = 2 = \frac{-a+2}{b-1} \Rightarrow a+b = 3$
 $\begin{cases} b = \frac{1}{2} \\ a = \frac{5}{2} \end{cases}$
 $b+c = 2 + \frac{1}{2} = \frac{5}{2}$

$f(n) = g(n) \Rightarrow \frac{(n+1)}{n-c} = \frac{(n-1)(n+1)}{n^2 - an + b} \Rightarrow$
 $\frac{n+1}{n-1} \Rightarrow c=1 \Rightarrow a=-2, b=1$
 $(n-1)^2 = n^2 - 2n + 1$
 $a+b+c = 0$

$f(1+n) = (1+n-3)^2(1+n-1) = (n-2)^2 n = n^3 - 4n^2 + 4n$
 $f(1-n) = (1-n-3)^2(1-n-1) = (-n-2)^2(-n) = -n^3 - 4n^2 - 4n$
 $\Rightarrow g(n) = -n^3 - 4n^2 - 4n = -1 \cdot n^3 = g(n)$

$$1-2n=-1 \Rightarrow n=1 \Rightarrow f(1) = f(-1)$$

$$1-2n=3 \Rightarrow n=-1 \Rightarrow f(-1) = f(1)$$

$$f(1) + f(-1) = 1 + 1 = 2$$

$$x + \sqrt{x} = 4 \Rightarrow x = 9$$

$$f(x + \sqrt{x}) + f(x) = 109 + 11 = 120$$

$$\frac{7+109+11}{3(1+109)}$$

$$\frac{(7+109+11)(1+109)}{3(1+109)}$$

$$\frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

$$f(n-1) = (n-1)^x + (n-1) = (n-1)(n^x + n + 1) + (n-1) = (n-1)(n^x + n + 2)$$

$$(n-1)(n^x + n + 2) \quad (1)$$

$$f(1-n) = (1-n)^x + (1-n) = (1-n)(1 + n^x + n) + (1-n) = (1-n)(n^x + n + 2)$$

$$(1-n)(n^x + n + 2) \quad (2)$$

$$(1) + (2) = (n^x + n + 2)(n-1 + 1-n) = 0$$

$$Df = \{n \in \mathbb{Z} \mid n \geq \frac{a}{2}\}$$

$$Dg = \{n \in \mathbb{Z} \mid n \leq \frac{b}{2}\} \Rightarrow Df \cap Dg = \{1\} \Rightarrow a = \frac{2}{3}, b = \frac{2}{3}$$

$$f(1) = \sqrt{1 - 2(\frac{2}{3})} + k - 1 = k - 1$$

$$g(1) = 1$$

$$\Rightarrow (f-g)(1) = -k - 1 = k$$

$$\Rightarrow k = -1$$

$$\star \text{ جواب } 2a - \frac{b}{2} - k = 2 - 1 + 1 = 2$$

$$Df \times g = Df \cap Dg = [-1, 1] ; Dg = [-1, +\infty) \Rightarrow Df = (-\infty, 1]$$

$$\Rightarrow m - n \geq 0 \Rightarrow n \leq m \Rightarrow m = 1$$

$$g(x) = x\sqrt{x}, f(x) = \sqrt{x^2 - x} + n = x + n$$

$$f-g(x) = x + n - x\sqrt{x} = 4\sqrt{x} \Rightarrow \sqrt{x^2 - x} - x = n$$

$$m + n = 1 + 1\sqrt{2} - 1 = 1 + \sqrt{2}$$