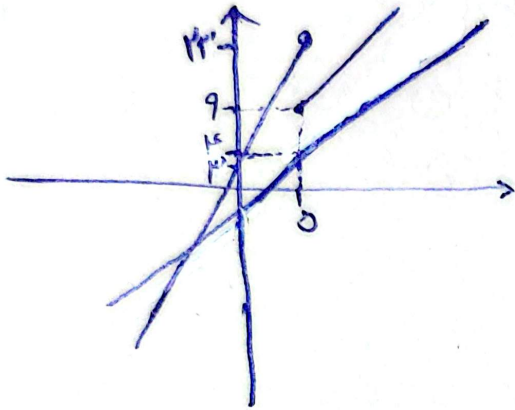


$$f(n) - n + 1 > 0 \rightarrow f(n) > n - 1$$

(1)

$$f(n) = \begin{cases} n - 1 & [n] > \varepsilon \rightarrow n > a \\ \varepsilon n + c & [n] \leq \varepsilon \rightarrow n < a \end{cases}$$



$$n - 1 = \varepsilon n + c \rightarrow n = \frac{-c}{\varepsilon}$$

$$D_y = \left[\frac{-c}{\varepsilon}, +\infty \right)$$

$$y = \frac{r\varepsilon n^5 + r^3 n^2 + 12n + c}{(\varepsilon n^2 + \varepsilon n + 1)^2} = \frac{r(rn + 1)^3}{((rn + 1)^2)^2} = \frac{r}{rn + 1}$$

$$rn + 1 \neq 0 \rightarrow n \neq \frac{-1}{r} \quad D_y = \mathbb{R} - \left\{ \frac{-1}{r} \right\}$$

$$n = 0 \rightarrow f(0) = g(0) \rightarrow \frac{-r}{-1} = a + c \rightarrow \boxed{c = r}$$

$$n = 1 \rightarrow f(1) = g(1) \rightarrow \frac{a+r}{1+b} = \frac{1+r}{c} \rightarrow a+r = r + cb \rightarrow \begin{cases} a - rb = 1 \end{cases}$$

$$n = -1 \rightarrow f(-1) = g(-1) \rightarrow \frac{-a+r}{-1+b} = \frac{-1+r}{c} \rightarrow -a+r = -1+b \rightarrow \begin{cases} -a - b = -c \end{cases}$$

$$b + c = \frac{1}{r} + r = \left(\frac{a}{r} \right)$$

$$\begin{aligned} -\varepsilon b &= -r \\ \hookrightarrow \boxed{b = \frac{1}{r}} \end{aligned}$$

$$\frac{n^r - 1}{n^r + an + b} = \frac{(n-1)(n+1)}{n^r + an + b} = \frac{n+1}{n-c} \quad (\Sigma)$$

مربع کامل است
 $n^r + an + b = (n-1)^r$
 $\begin{cases} a = -r \\ b = 1 \end{cases}$
 $n - c = n - 1 \rightarrow c = 1$
 $a + b + c = -r + 1 + 1 = 0$

$$\begin{aligned} g(n) &= f(n+1) + f(1-n) = ((n+1-r)^r (n+1-1)) + ((1-n-c)^r (1-n-1)) \\ &= ((n-r)^r (n)) + ((-n-r)^r (-n)) = n(n^r - rn + r - n^r - rn - r) \\ &= \boxed{-1n^r} \end{aligned} \quad (\Theta)$$

$$f(-1) + f(c) = r + 1 = 0 \quad (\delta)$$

$$f(-1) \rightarrow f(1-rn) = f(-1) \rightarrow 1-rn = -1 \rightarrow n = 1$$

$$\underline{n=1}, \quad r\sqrt{1} = 0 \quad (\Sigma)$$

$$f(c) \rightarrow f(1-rn) = f(c) \rightarrow 1-rn = c \rightarrow n = -1$$

$$\underline{n=-1}, \quad r\sqrt{-1} + r = 0 \quad (\Theta)$$

$$f(y) = f(n + \sqrt{r}) \rightarrow y = 2c + \sqrt{r} \xrightarrow{\sqrt{r}=t} t^r + t - y = 0 \quad (\checkmark)$$

$$\rightarrow (t+c)(t-r) = 0 \begin{cases} t = -c \sqrt{r} \\ t = r \rightarrow \sqrt{r} = r \rightarrow n = r \end{cases}$$

$$f(y) = (r)^r + r \times r = 2r^2 \quad (\Theta)$$

$$f(n-1) + f(1-n) = f(n-1) + f(-(n-1)) \xrightarrow{n-1=t} f(t) + f(-t) = \textcircled{0} \quad \textcircled{1}$$

$$f(n) + f(-n) = (n^c + n) + (-n^c - n) = 0 \xrightarrow{n=t} f(t) + f(-t) = 0 \quad \uparrow$$

$$D_f \cap D_g = \{1\}$$

①

$$b - \varepsilon n \geq 0 \rightarrow n \leq \frac{b}{\varepsilon} \rightarrow \frac{b}{\varepsilon} = 1 \rightarrow b = \varepsilon$$

$$\varepsilon n - \varepsilon a \geq 0 \rightarrow n \geq \frac{\varepsilon a}{\varepsilon} \rightarrow \frac{\varepsilon a}{\varepsilon} = 1 \rightarrow a = \frac{\varepsilon}{\varepsilon}$$

$$n=1 \rightarrow r f_0 - g(1) = r(k-1) - rk = -k - r$$

$$-k - r = k \rightarrow k = -1$$

$$ra - \frac{b}{r} - k = r \times \frac{\varepsilon}{\varepsilon} - \frac{\varepsilon}{r} - (-1) = \textcircled{r}$$

$$D_g \cap D_f = [-1, v]$$

①

$$r n + r \geq 0 \rightarrow n \geq -1 \rightarrow D_g = [-1, +\infty) \rightarrow D_f = (-\infty, v]$$

$$m - n \geq 0 \rightarrow m \geq n \rightarrow m = v$$

$$f(c) - g(c) = r + n - r\sqrt{r} = 4\sqrt{r} \rightarrow n = 4\sqrt{r} - r$$

$$m + n = v + 4\sqrt{r} - r = \boxed{v + 4\sqrt{r}}$$