

1
$$\begin{cases} f(x) - x f(-x) = 2|x| + 1 \\ f(x) + x f(-x) = 2|x| + 1 \end{cases} \xrightarrow{x \rightarrow -x} \begin{cases} -x f(x) + x^2 f(-x) = 2|x| - x \\ f(x) + x f(-x) = 2|x| + 1 \end{cases} \Rightarrow f(x) = \begin{cases} \frac{-x^2 + x + 1}{x^2 + 1} & x > 0 \\ 1 & x = 0 \\ \frac{x^2 - x + 1}{x^2 + 1} & x < 0 \end{cases}$$

$$\Rightarrow (x^2 + 1)f(x) = 2|x|x - x + 2|x| + 1$$

2
$$f(0) : 2f(0) = 4 \Rightarrow f(0) = 2 \Rightarrow f(x) = x^2 - 3x + 2 \Rightarrow f(x^2) = x^4 - 3x^2 + 2$$

3
$$yx - y = x^2 + x + 2 \Rightarrow x^2 + (1-y)x + (2+y) = 0$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow y = \frac{1 \pm \sqrt{(1-y)^2 - 4(2+y)}}{2}$$

$$\Rightarrow \begin{cases} (1-y)^2 - 4(2+y) \geq 0 \\ y^2 + 1 - 2y - 8 - 4y \geq 0 \\ y^2 - 6y - 7 \geq 0 \end{cases} \Rightarrow \begin{cases} -1 \leq y \leq 7 \\ y \leq -1 \text{ or } y \geq 7 \end{cases} \Rightarrow R_f = [-1, 7]$$

الف)
$$\frac{-b}{2a} = \frac{\Delta}{4} \Rightarrow 2x \frac{2\Delta}{16} - \Delta x \frac{\Delta}{4} + 1 = \frac{2\Delta - \Delta^2 + 4}{4} = \frac{-14}{4} \Rightarrow R_f = [-\frac{14}{4}, +\infty)$$

ب)
$$\frac{-b}{2a} = \frac{4}{-2} = -2 \Rightarrow -9 + 19 - \Delta \leq 2 \Rightarrow \Delta \geq 8 \Rightarrow R_f = [2, \sqrt{2}]$$

ج)
$$R = \mathbb{R} \xrightarrow{\text{اصل رادیکال}} R = [0, +\infty)$$

د)
$$R = \mathbb{R} = (-\infty, +\infty) \Rightarrow R_f = \mathbb{R}$$

الف)
$$R = \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

ب)
$$R = \mathbb{R} - \{ \frac{3}{2} \} \xrightarrow{\text{اصل رادیکال}} R_f = [2, +\infty) - \left\{ \frac{3}{2} \right\}$$

ج)
$$y = \frac{x^2 + 4 + 1}{\sqrt{x^2 + 4}} = \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}} \xrightarrow{\min(\sqrt{x^2 + 4}) = \sqrt{4}} R_f = \left[\frac{4\sqrt{5}}{5}, +\infty \right)$$

د)
$$y = x^2 + 4 + \frac{1}{x^2 + 4} \xrightarrow{\min(x^2 + 4) = 4} R_f = \left[\frac{1}{4}, +\infty \right)$$

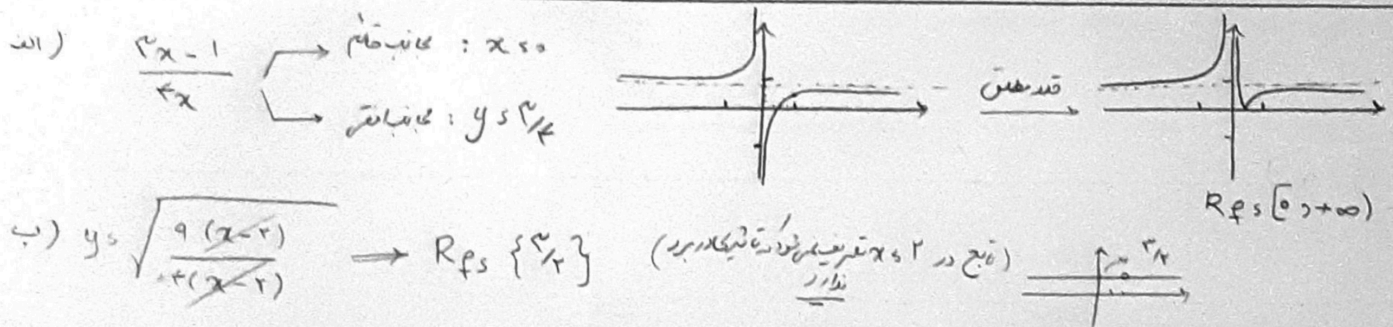
$$yx^2 - y = x^2 + 4 \Rightarrow (y-1)x^2 = 4 + y \xrightarrow{y \neq 1} x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\pm \sqrt{4(y-1)(y+4)}}{2(y-1)}$$

$$\Delta = 4(y-1)(y+4) \geq 0 \Rightarrow (y-1)(y+4) \geq 0 \Rightarrow y \leq -4 \text{ or } y \geq 1$$

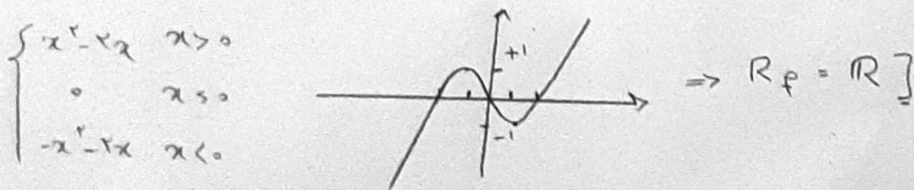
$$R_f = (-\infty, -4] \cup [1, +\infty)$$

$$\sin \omega \begin{cases} \min -1 \rightarrow f(-1) = -1/2 \\ \max 1 \rightarrow f(1) = -3 \end{cases} \xrightarrow{\text{کنج مندرجه}} R_f = [-3, -1/2]$$

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