

$$x \cdot f(-x) - x^r f(x) = r|x| + 1$$

$$+ x^r f(-x) + f(x) = \frac{r|x| + 1}{1+x^r}$$

- $x \cdot x^r f(x)$
 $\rightarrow$ $x^{r+1} f(x)$

$$f(x) + x^r f(x) = r|x| - r|x| + 1 - x$$

$$f(x)(1+x^r) = r|x|(1-x) + 1-x$$

$$f(x) = \frac{r|x|(1-x) + 1-x}{1+x^r}$$

$$\left\{ \begin{array}{l} \frac{r|x|(1-x) + 1-x}{1+x^r} \quad x \geq 0 \\ = \frac{rx - rx^r + 1 - x}{1+x^r} = \frac{-x^r + x + 1}{1+x^r} \\ -rx + x^r + 1 - x \quad x < 0 \\ = \frac{-x^r - rx + 1}{1+x^r} \end{array} \right.$$

$$f(x) + f(0) = x^r - rx + 1 \xrightarrow{x=0} r f(0) = 1 \Rightarrow f(0) = \frac{1}{r}$$

$$x \Rightarrow x^r \rightarrow f(x^r) + \frac{1}{r} = x^r - rx^r + 1$$

$$\Rightarrow f(x^r) = x^r - rx^r + \frac{1}{r}$$

$$y = \frac{x^r + x + r}{x-1} \Rightarrow x^r + x + r = y(x-1) \Rightarrow x^r + (1-y)x + r+y = 0$$

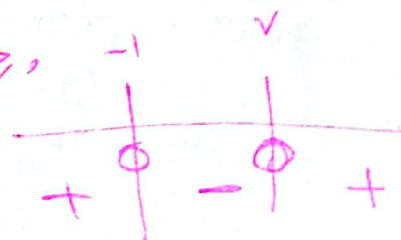
$$x = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4x(r+y)}}{2}$$

$$= \frac{-1+y \pm \sqrt{y^2 - ry + 1 - 4 - 4y}}{2}$$

$\Delta \geq 0$

$$y^2 - ry - 1 \geq 0, (y-r)(y+1) \geq 0$$

$$R_f:]R - (-1, r)$$



الف) $y = 2x^2 - 8x + 1$

$R_f: \left[\frac{-14}{2}, +\infty \right)$

$x_{min} = \frac{-b}{2a} = \frac{4}{2}$

$y_{min} = 2\left(\frac{4}{2}\right)^2 - 8\left(\frac{4}{2}\right) + 1 = \frac{2 \cdot 16}{2} - \frac{32}{2} + \frac{2}{2} = \frac{-14}{2}$

$\frac{-20}{2}$

ب) $y = \sqrt{-x^2 + 4x - 8}$

$R_f = (-\infty, 4] \Rightarrow \sqrt{} \cdot [0, 4]$

$x_{max} = \frac{-b}{2a} = \frac{-4}{-2}$

$y_{max} = -9 + 11 - 2 = 0$

ج) $y = \sqrt{x^2 - 4x^2 + 2x + 1}$ $\Rightarrow R_f = [0, +\infty)$

د) $\log(x^2 - 4x^2 + 2x + 1) > 0$ $\log(0, +\infty) = \mathbb{R}$ $(R_f \subset \mathbb{R})$

هـ) $\frac{3x+1}{2x-4}$ $\Rightarrow \mathbb{R} - \left\{ \frac{4}{3} \right\}$

د

و) $y = \sqrt{\frac{4x+1}{x-2}}$ $\Rightarrow \mathbb{R} - \left\{ \frac{1}{4} \right\}$ $[0, +\infty) - \left\{ \frac{1}{4} \right\}$

ز) $y = \frac{x^2 + 4}{\sqrt{x^2 + 4}}$ $= \frac{x^2 + 4 + 1}{\sqrt{x^2 + 4}}$ $= \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}}$

$= \sqrt{4} + \frac{1}{\sqrt{4}} = \frac{4+1}{2} = \frac{5}{2} = \frac{5}{2} \times \frac{\sqrt{4}}{\sqrt{4}} = \frac{5\sqrt{4}}{2}$

$R_f = \left[\frac{5\sqrt{4}}{2}, +\infty \right)$

$\sqrt{x^2 + 4} = 1$ $\Rightarrow x^2 + 4 = 1$ $\Rightarrow x^2 = -3$ $\Rightarrow$ لا يوجد حلا حقيقي

$\sqrt{x^2 + 4} = \sqrt{4}$

به نام خداوندگار

د) $y = \frac{x^2 + \frac{1}{x}}{x^2 + 2} - 2 \Rightarrow a + \frac{1}{a}$ $a > 0$ $\frac{x^2 + 2}{x^2 + 2} = 1$ $a_2 = 2$ و $a_1 = 2$

$2 + \frac{1}{2} = \frac{5}{2} \Rightarrow R_f = \left(\frac{5}{2}, +\infty \right) - 2 = \left[\frac{1}{2}, +\infty \right) = IR$

$y = \frac{x^2 + 2}{x^2 - 1}$ روش اصلی $y = \frac{x^2 + 2}{x^2 - 1} \Rightarrow y(x^2 - 1) = x^2 + 2 \Rightarrow yx^2 - y = x^2 + 2 \Rightarrow yx^2 - x^2 = y + 2 \Rightarrow x^2(y - 1) = y + 2$ ④

$R_f = (-\infty, -4] \cup (3, +\infty)$ $\frac{-2}{+} \quad \frac{3}{-} \quad \frac{+}{+}$

روش تغییر $x^2 \rightarrow \min 0$ $\frac{2}{-1} = -2$
 $\rightarrow \max +\infty$ $\frac{2 + \infty}{\infty^2 - 1} \rightarrow \frac{+\infty}{+\infty} = 1$ نقطه = 3

مخرج به ازای دو -1 صفر شود
 در نقطه صاف این دو باره
 جواب است

$R_f = (-\infty, -4] \cup (3, +\infty)$

بهمین روش $+\infty$ یک صفت است باز هم صفر باز

$f(x) = \frac{2 \sin x + 2}{\sin x - 3}$ $\min \sin x = -1$ $\frac{2(-1) + 2}{-1 - 3} = \frac{0}{-4} = 0$ ⑤
 $\max \sin x = 1$ $\frac{2(1) + 2}{1 - 3} = \frac{4}{-2} = -2$ ⑥

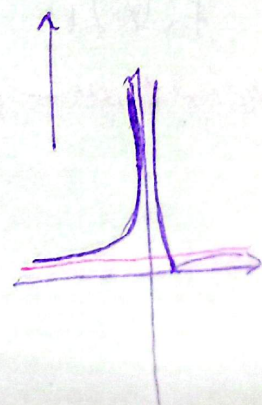
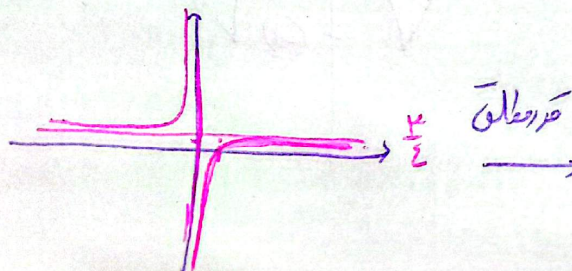
$R_f = [-2, 0]$

الف) $y = \frac{x^2 - 1}{x^2}$ مردمان ⑦ $R = IR - \left\{ \frac{1}{2} \right\} \Rightarrow R_f = [0, +\infty)$ ⑧

صورت $\frac{x^2 - 1}{x^2} = \frac{x^2}{x^2} - \frac{1}{x^2} = 1 - \frac{1}{x^2}$ مردمان نقطه

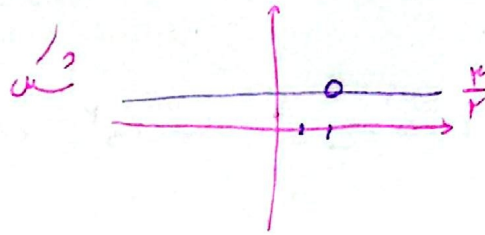
② رسم

$x = 1 \Rightarrow \frac{x^2 - 1}{x^2} = \frac{1 - 1}{1} = 0$



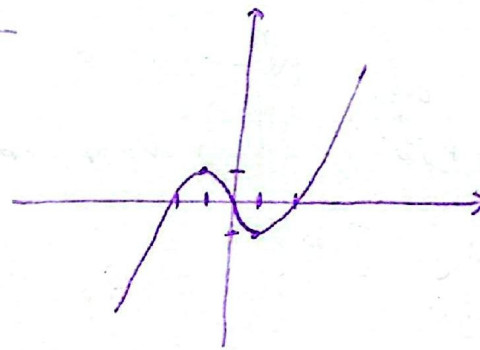
بنا کردن دایره

$$y = \sqrt{\frac{9x-18}{-11+2x}} = \sqrt{\frac{9(x-2)}{2(x-2)}} = \sqrt{\frac{9}{2}} \Rightarrow \boxed{R_f = \frac{3}{\sqrt{2}}}$$



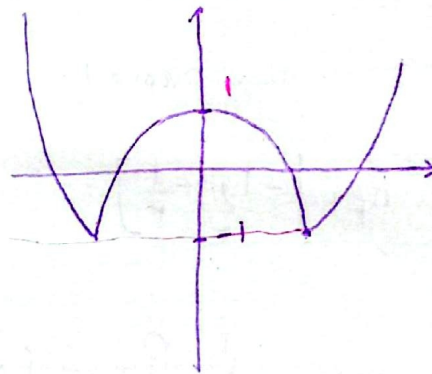
$$y = x|x| - 2x$$

$-x^2 - 2x$	$x^2 - 2x$
$-x(x+2)$	$x(x-2)$
$\frac{x}{x+1} = -1$	$\frac{x}{x-2} = 1$
$-1 + 2 = 1$	$1 - 2 = -1$



$R_f = \mathbb{R}$

$$y = |f(|x|) + 1|$$



$f(x) + 1$

$$y = |f(|x|) + 1|$$

$f(|x|) + 1$ باز هم همین است
یعنی یک تصویر است به معنی که قرینه است

