

$n \rightarrow$
 $f(-x) - x f(x) = x|x| + 1$ $f(x) = ?$

$n = -n$
 $f(x) + x f(-x) = x|-x| + 1 \Rightarrow$
 $x f(-x) + f(x) = x|x| + 1$

$f(x) + x^2 f(x) = x|x| - x|x| - x + 1$

$\Rightarrow f(x) = \frac{x|x| (1 - x) - x + 1}{1 + x^2} = \frac{x|x| - 2x|x| - x + 1}{1 + x^2}$

$f(x) = \begin{cases} \frac{-x^2 + x + 1}{1 + x^2} & ; x \geq 0 \\ \frac{x^2 - x + 1}{x^2 + 1} & ; x < 0 \end{cases}$

$f(x) + f(x) = x^2 - x^2 + 1 \xrightarrow{n=x} x f(x) = 1 \rightarrow f(x) = \frac{1}{x} \rightarrow f(x) = x^2 - x^2 + 1$
 $f(x^2) = x^2 - x^2 + 1$

$y = \frac{x^2 + x + 1}{x - 1} \rightarrow y(x - 1) - x^2 - x = 1 \rightarrow -x^2 + (y - 1)x - y + 1 = 0$

$x = \frac{-y + 1 \pm \sqrt{y^2 - 4y + 1}}{-2} \Delta \geq 0 \rightarrow y^2 - 4y + 1 \geq 0 \rightarrow R = (-\infty, -1] \cup [1, +\infty)$

و) $y = x^2 - 2x + 1 \rightarrow A_{min} \Big|_{\frac{0}{x}} \Big|_{x = \frac{2x}{2} = 1} \rightarrow y = \frac{2}{1} - \frac{2 \cdot 2}{1} + 1 = -1 \rightarrow R = [-1, +\infty)$

ب) $y = \sqrt{-x^2 + 4x - 8} \rightarrow A_{max} \Big|_{x^2} \Big|_{-9 + 1n - 8 = 4} \rightarrow R_{y'} = (-\infty, 4]$
 $R_y = [0, 4]$
 $R_y = \sqrt{R_{y'}}$

ج) $y = \sqrt{x^2 - 4x + 4n + 1}$ $R_{y'} = \mathbb{R} \rightarrow R_y = \sqrt{R_{y'}} = [0, +\infty)$ -4

د) $y = \lg(x^2 - 4x + 4n + 1)$ $R = \mathbb{R} \rightarrow$ $\begin{matrix} \text{دستگاه} \\ \text{معادلات} \end{matrix} (0, +\infty) \rightarrow R_y = \mathbb{R}$

هـ) $y = \frac{x_{n+1}}{x_n - 4} \rightarrow R = \mathbb{R} - \left\{ \frac{x}{4} \right\}$ -5

و) $y = \sqrt{\frac{x_{n+1}}{x_n - 4}}$ $R_{y'} = \mathbb{R} - \{4\} \rightarrow R_y = [0, +\infty) - \{2\}$

ز) $y = \frac{x^2 + 4}{\sqrt{x^2 + 4}} = \frac{x^2 + 4}{\sqrt{x^2 + 4}} + \frac{1}{\sqrt{x^2 + 4}} = a + \frac{1}{a} \rightarrow y_{\min} = \frac{4}{\sqrt{4}} + \frac{1}{\sqrt{4}} = \frac{4\sqrt{4}}{4}$

$R_y = \left[\frac{4\sqrt{4}}{4}, +\infty \right)$

ح) $y = x^2 + \frac{1}{x^2 + 4}$ $\begin{matrix} \text{محدود} \\ \text{محدود} \end{matrix} 0 + \frac{1}{4} = \frac{1}{4}$
 $\begin{matrix} \text{محدود} \\ \text{محدود} \end{matrix} \left[x^2_{\max} \right] \rightarrow \frac{+\infty}{+\infty} + \frac{1}{+\infty + 4} = +\infty$

$\rightarrow R_y = \left[\frac{1}{4}, +\infty \right)$

$y = \frac{3x^2 + 4}{x^2 - 1}$ (14)

دستگاه معادلات $x^2 = \frac{y+4}{y-3} \rightarrow x = \pm \sqrt{\frac{y+4}{y-3}}$ $\frac{-4}{+} \frac{3}{-} \frac{+}{+}$

$\rightarrow R_y = (-\infty, -4] \cup (3, +\infty)$

تکرار $x^2_{\min} = 0 \rightarrow y = -4$ $\frac{x^2 - 1}{\text{محدود}}$ $R = (-\infty, -4] \cup (3, +\infty)$
 $x^2_{\max} = +\infty \rightarrow y = 3$

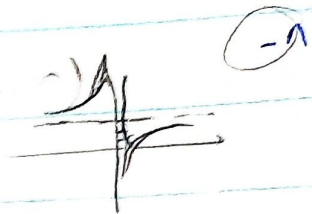
$$f(x) = \frac{x \sin x + 4}{\sin x - 2} \quad \xrightarrow{\max \sin x = 1} \quad \sin x = 1 \rightarrow y = \frac{4}{-1} = -4$$

و نیز
 $\sin x = 0$

$$\xrightarrow{\min \sin x = -1} \quad \frac{4}{-3} = -\frac{4}{3}$$

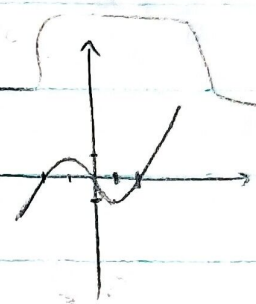
$$\rightarrow R = \left[-4, -\frac{4}{3}\right]$$

الف) $y = \left| \frac{x^2 - 1}{x^2} \right| \rightarrow R_{y'} = |R| \rightarrow R_y = [0, +\infty)$

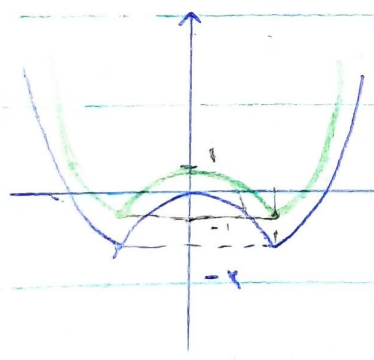


ب) $y = \sqrt{\frac{4x - 1}{-x + 2}}$ $\rightarrow y = \sqrt{\frac{4(x-0.25)}{-(x-2)}}$ $\xrightarrow{x \rightarrow 2^-} \infty$ $\xrightarrow{x \rightarrow 0.25^+} 0$ $\rightarrow R_y = \{y \mid y \geq 0\}$

$y = a|x| - x^2 \xrightarrow{x \geq 0} y = x^2 - x^2$
 $\xrightarrow{x < 0} y = -x^2 - x^2$

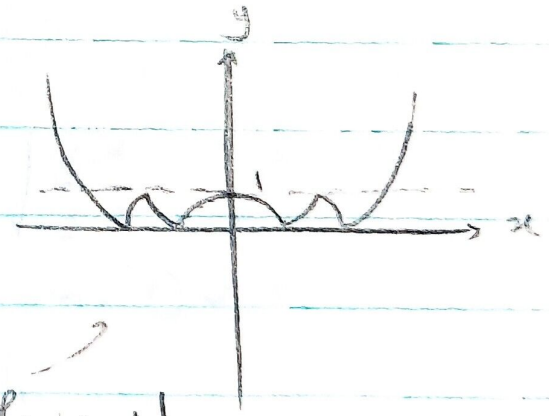


$$R_y = \mathbb{R}$$



$$y = f(x)$$

$$y = |f(x)| + 1$$



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Somehow we can find the range of the function

$$\frac{y - 1}{2} = \sqrt{y - 1}$$

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