

$f(-x) - xf(x) = 2 x + 1 \Rightarrow f(-x) = 2 x + 1 + xf(x)$ $\rightarrow \text{با جای } x \Rightarrow f(x) + xf(-x) = 2 x + 1 \xrightarrow{f(-x) = 2 x + 1 + xf(x)} f(x) + x(2 x + 1 + xf(x)) = 2 x + 1$ $\rightarrow f(x) + 2 x x + x + x^2 f(x) = 2 x + 1 \rightarrow f(x)(x^2 + 1) = 2 x (1-x) + 1 - x$ $\rightarrow f(x) = \frac{2x(1-x) + 1 - x}{x^2 + 1}$	۱
$x=0 \rightarrow 2f(0) = 4 \rightarrow f(0) = 2$ $x^2 \in x \Rightarrow f(x^2) + 2 = (x^2)^2 - 2(x^2) + 4 \rightarrow f(x^2) = x^4 - 2x^2 + 2$	۲
$y(x-1) = x^2 + x + 1 \Rightarrow yx - y = x^2 + x + 1 \Rightarrow x^2 + x(1-y) + y + 1 = 0$ $\rightarrow x = \frac{(y+1) \pm \sqrt{(1-y)^2 - 4(y+1)}}{2} \rightarrow \Delta \geq 0 \rightarrow 1 + y^2 - 2y - 4 - 4y \geq 0$ $\rightarrow y^2 - 4y - 3 \geq 0 \rightarrow (y-5)(y+1) \geq 0 \rightarrow \frac{-1}{+} \frac{5}{-} \rightarrow R_f = (-\infty, -1] \cup [5, +\infty)$	۳
$x_0 = \frac{a}{f} \rightarrow y_{min} = 2\left(\frac{x_0}{14}\right) - 2\left(\frac{a}{f}\right) + 1 \Rightarrow y_{min} = \frac{2a}{14} - \frac{2a}{f} + \frac{1}{f} = -\frac{14}{f}$ $\rightarrow R = \left[-\frac{14}{f}, +\infty\right)$ $x_{max} = \frac{-4}{-r} = 2 \rightarrow y_{max} = f \rightarrow R = \sqrt{(-\infty, f)} \rightarrow R = [0, 2]$ $R_y = \sqrt{R} \rightarrow R = [0, +\infty)$ $R_y = \log(R) \rightarrow (-\infty, +\infty)$	۴
$R = \mathbb{R} - \left\{\frac{a}{e}\right\} \Rightarrow R = \mathbb{R} - \left\{\frac{2}{e}\right\}$ $R_f = [0, +\infty) - \left\{1\right\}$ $\frac{(\sqrt{x^2+3})^2 + 1}{\sqrt{x^2+3}} \rightarrow \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \xrightarrow{\text{مقدار مثبت}} \left[\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty\right) \rightarrow \left[\frac{4\sqrt{3}}{3}, +\infty\right)$ $\xrightarrow{\text{تفاضل}} \left[-f + \frac{1}{f}, +\infty\right) \rightarrow \left[\frac{1}{f}, +\infty\right)$	۵

$yx^r - y = x^r + f \rightarrow x^r(y - 1) = f + y \rightarrow x^r = \frac{f+y}{y-1} \rightarrow x = \pm \sqrt{\frac{f+y}{y-1}}$ $\rightarrow R_f = (-\infty, -f] \cup (1, +\infty)$ $x^r \rightarrow x = +\infty \rightarrow 1$ $x^r \rightarrow x = 0 \rightarrow -f$ $\Rightarrow (-\infty, -f] \cup (1, +\infty)$	روس اصلی: $\frac{-f}{1-f} = \frac{1}{1-f}$ به روش دیگر: $\frac{1}{1-f}$ ٦
$\sin x \rightarrow -1 \rightarrow \frac{1}{-1} = -1$ $\sin x \rightarrow 1 \rightarrow \frac{1}{1} = 1$ $\Rightarrow R_f = [-1, 1]$	٧
$x \neq 0, y > 0 \rightarrow R_y = [0, +\infty) - \left\{ \frac{1}{x} \right\}$ $\sqrt{\frac{9x-1}{x-1}} = y \rightarrow y = \sqrt{\frac{9(x-1)}{x-1}} \Rightarrow R_f = \left\{ \frac{1}{x} \right\}$	الف) ٨
$x > 0 \rightarrow x(x-2)$ $x < 0 \rightarrow x(-x-2)$ $\rightarrow R_y = \mathbb{R}$	٩
$f(x) + 1$ $y = f(x) + 1$ جواب:	① $f(x)$ ② $f(x) + 1$ ③ $y = f(x) + 1$ ١٠