

① $f_{\text{max}} = \frac{(1-n)(2n+1)}{1+n^2}$ ②

$-ax f(-a) - a f(a) = 2|a| + 1 \quad f(a) = ?$

$f(a) + a f(-a) = 2|a| + 1$

$-a f(-a) + a^2 f(a) = -2a|a| - a$

$f(a) + a f(-a) = 2|a| + 1$

$n^2 f(a) + f(a) = -2n|a| - a + 2|a| + 1$

$f(a)(n^2+1) = -2|a| - a + 2|a| + 1 \Rightarrow f(a) = \frac{-2|a| - a + 2|a| + 1}{n^2+1}$

فکتور ↑

$f(n^2) = 0 \quad f(a) + f(0) = a^2 - 2an + 4$

$f(n^2) = n^2 - 2an + 4$

$a=0 \rightarrow f(0) + f(0) = 4 \Rightarrow f(0) = 2 \rightarrow f(a) + 2 = n^2 - 2an + 4 \rightarrow f(a) = a^2 - 2an + 2$

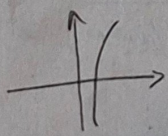
$y = \frac{a^2 + a + 1}{a - 1} \quad ya - y = a^2 + a + 1 \rightarrow n^2 + (1-y)a + (1+y) = 0 \rightarrow a = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \Delta = b^2 - 4ac$

$(1-y)^2 - 4(1+y) \geq 0 \rightarrow y^2 - 2y + 1 - 4 - 4y \geq 0 \rightarrow y^2 - 4y - 3 \geq 0$
 $\Rightarrow Ry = (-\infty, -1] \cup [3, +\infty)$

$y = 2a^2 - 2a + 1 \quad \begin{cases} u_s = \frac{-b}{2a} = \frac{1}{2} \\ y_s = 2(\frac{1}{2}) - 2(\frac{1}{2}) + 1 = \frac{1}{2} - 1 + 1 = \frac{1}{2} \end{cases} \rightarrow R_f = [-\frac{1}{2}, +\infty)$

$y = \sqrt{-a^2 + 4a - 2}$

$\begin{cases} u_s = \frac{-b}{2a} = \frac{-4}{-2} = +2 \\ y_s = 4 \end{cases} \Rightarrow \sqrt{(-\infty, 4]} \Rightarrow Ry = [-2, 4]$



$y = \sqrt{a^2 - 2a^2 + 4a + 1} \rightarrow Ry = [0, +\infty)$
 $Ry = \sqrt{R}$

$y = \log(a^2 - 2a^2 + 4a + 1) \rightarrow Ry = (-\infty, +\infty)$
 $\log(0, +\infty) \Rightarrow Ry = (-\infty, +\infty)$

$y = \frac{a^2 + 1}{a^2 - 4} \rightarrow Ry = R - \left\{ \frac{1}{4} \right\}$

$y = \frac{a^2 + 1}{\sqrt{a^2 + 1}} = \frac{\sqrt{a^2 + 1}}{1} + \frac{1}{\sqrt{a^2 + 1}} \Rightarrow Ry = \left[\frac{1}{\sqrt{2}}, +\infty \right)$
 $\min = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$y = \sqrt{\frac{a^2 + 1}{a - 2}} \rightarrow \sqrt{R - \left\{ \frac{1}{2} \right\}} \rightarrow Ry = [0, +\infty) - \left\{ \frac{1}{2} \right\}$

$y = \frac{a^2 + 1}{a^2 + 1} - 1 \Rightarrow Ry = \left[\frac{1}{2}, +\infty \right)$
 $\min = 1 + \frac{1}{2} - 1 = \frac{1}{2}$

$y = \frac{2a^2 + 1}{a^2 - 1} \xrightarrow{\text{ob.}} a^2 = \frac{-y-1}{y-2} = \frac{y+1}{y-2} \rightarrow a = \pm \sqrt{\frac{y+1}{y-2}}$
 $\Rightarrow Ry = (-\infty, -1] \cup \left[\frac{3}{2}, +\infty \right)$

$a^2 = 0 \rightarrow y = \frac{1}{-1} = -1$

$a^2 = +\infty \rightarrow y = \frac{2}{1} = 2$

حرف متوازن می شود (if a = ±1) پس خارج از محدوده است و چون
 به ازای +∞ به دست می آید پس باز است.

$\hookrightarrow Ry = (-\infty, -1] \cup [2, +\infty)$

$$f(x) = \frac{2 \sin ax + c}{\sin ax - 3}$$

$$\begin{matrix} \xrightarrow{\sin ax = +1} \\ \xrightarrow{\sin ax = -1} \end{matrix}$$

$$\frac{2+c}{-3} = \frac{4}{-3} = -\frac{4}{3}$$

$$\frac{-2+c}{-1-3} = \frac{-2+c}{-4} = -\frac{1}{4}$$

$$\Rightarrow R_y = [-\frac{4}{3}, -\frac{1}{4}]$$

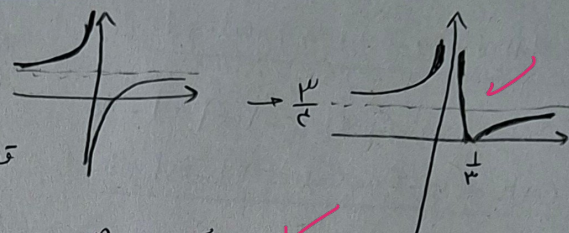
همه خروجیها صفر شدن ندارند چون
صفر وقتی $\sin ax$ برابر 3 می شود پس
این عدد است

(2) (1)

$$y = \left| \frac{3a-1}{a} \right|$$

$$\text{دست چپ} = 0 \rightarrow$$

$$\text{دست راست} = \frac{a}{c} = \frac{3}{c}$$



(2) (1)

قدر مطلق در هر دو طرف $f(x)$ است پس y متغیرها را یاد کرده و آنجا که یاد کردیم را ترسیم کنیم

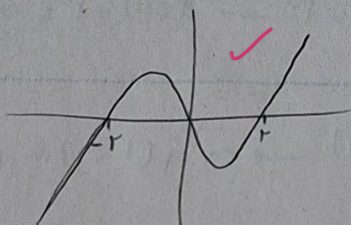
$$\frac{3a-1}{a} = 0 \rightarrow 3a-1=0 \rightarrow a=\frac{1}{3}$$

$$\Rightarrow R_y = [0, +\infty)$$

$$y = \sqrt{\frac{4a+11}{-1+2a}} = \sqrt{\frac{4(a-2)}{2(a-2)}} \Rightarrow y = \sqrt{\frac{4}{2}} = \frac{2}{\sqrt{2}} \Rightarrow R_y = \left\{ \frac{\sqrt{2}}{2} \right\}$$

$$y = a|a| - 2a$$

$$\begin{array}{l|l} -a^2 - 2a & a^2 - 2a \\ \hline a(-a-2) & a(a-2) \\ \downarrow & \downarrow \\ a = -2 & a = 2 \end{array}$$

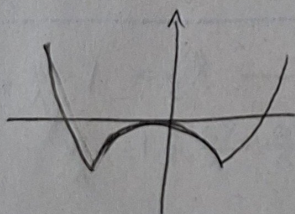


$$R_y = \mathbb{R}$$

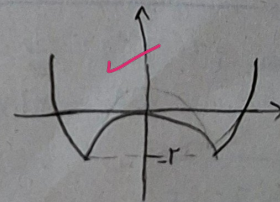
(2) (9)

$$y = |f(|a|) + 1|$$

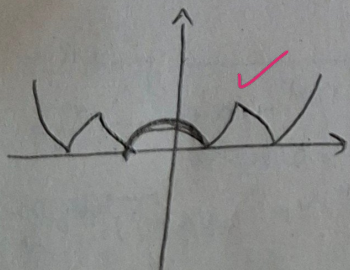
(2) (1)



همه ضرایب
آنها منفی نیستند

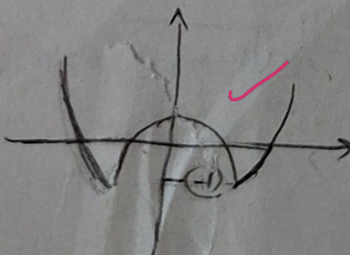


(2) (10)



$$\Rightarrow R_y = [0, +\infty)$$

همه ضرایب
آنها منفی نیستند



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